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New H_∞ Filter Design Approach for Time-Delay Fuzzy-Model-Based System under Imperfect Premise Matching

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***Abstract*—The paper concerns the issue of H_∞ fuzzy filter design for T-S fuzzy-model based system involving time-delay. In the light of the Lyapunov stability theory, improved stability criteria for the filtering system are developed, and corresponding H_∞ fuzzy filter design approaches are proposed as well, all the derived results are given as the feasibility of LMIs. Besides, the novel imperfect premise matching methodology is adopted to derive the H_∞ fuzzy filter, which allows the H_∞ fuzzy filter and the T-S fuzzy model have distinct membership functions and different number of fuzzy rules, such design can provide larger design flexibility and better robustness property. Finally, two numerical examples are offered to clarify the validity and the superiority of the designed method.**

***Keywords*—T-S fuzzy model, H_∞ filter design, time-varying delay, stability analysis, imperfect premise matching**

I. Introduction

Filtering plays a significant role in signal processing, last decades the filtering technology developed rapidly, various methodologies about filter design have been proposed, such as H_2 filter and H_∞ filter [1], etc. To mentions a few, the authors of [2] presented an H_∞ filter design approach for discrete systems involving some different time-delays. The authors of [3] investigate nonlinear systems relating to time-varying delay, and they developed novel H_∞ filter design approach in their paper. Compared with other filters, the main merit of the H_∞ filter is that it can deal systems with uncertain parameters and have no specific requirement for the external noises.

Besides, our research subject in this paper is nonlinear system, which is commonly repersented as T-S fuzzy model[4] in fuzzy control theory. When deriving the stability condition for the TSFMB system, the Lyapunov-Krasovskii functional (LKF) method is often adopted, which can obtain the results in view of the feasibility of LMIs. Usually the stability conditions derived with LKF method are conservative, to reduce the conservatism, researchers have proposed various methods: triple integral/summation terms [5] were introduced to make the LKFs more appropriate; the Wirtinger-based inequality [6], the free-matrix-based inequality were introduced to obtain more accurate bound of the integral terms. In this paper, both the T-S fuzzy model and the Lyapunov stability theory will be employed.

When designing H_∞ fuzzy filter for TSFMB system, the PDC method [7] is usually adopted, which assumes that the fuzzy filter and the T-S fuzzy model enjoy the uniform membership functions and the uniform number of rules. The PDC method can facilitate the filter design, however, the PDC method will also limit the flexibility to choose the membership functions for the fuzzy filter. To address such problems, the imperfect premise matching technique was proposed [8]. Contrary to the PDC method, the imperfect premise matching methodology allows the fuzzy H_∞ filter and T-S fuzzy model bear distinct membership functions and distinct number of fuzzy rules, and in recent years some new results about imperfect premise matching have been developed [9]. In this paper we will adopt the imperfect premise matching method to design H_∞ filter.

Moreover, the nonlinear system we investigate in this paper involves time-delay, which is a common phenomenon in complex nonlinear systems can pose instability and deteriorate the performance of the systems, therefore, the research about time-delay is of great practical and theoretical significance. And over the last decades, many significant results have been derived [10]. Specially, in 2000, Cao and Frank [11] first employed the T-S fuzzy model to describe the describe the dynamics of the nonlinear system involving time-delay.

From the discussions above, we can conclude that when designing fuzzy H_∞ filter for TSFMB systems with time-delay, the design flexibility will be limited with conventional PDC method, and the derived stability conditions are conservative with LKF method. Therefore, we aim to investigate an improved fuzzy H_∞ filter design method which is less conservative, and the novel imperfect premise matching method will be adopted. Besides, the structure of this paper is arranged as follows: Section II provides related preliminaries about T-S fuzzy model and fuzzy H_∞ filter. In Section III, the system stability is studied, and the corresponding H_∞ filter design methods are presented. Besides, in Section IV, some numerical examples are offered to clarify the validity and superiority of the designed approach in this paper. Last, Section V presented the conclusion of the paper.

II. Preliminaries

A T-S fuzzy-model-based filtering system involving time-varying delay is considered

Plant Rule:

Establish a p-rule polynomial fuzzy model to represent the dynamics of the nonlinear filtering system.

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{p} \omega_i(x(t))[A_i x(t) + A_{di}x(t-d(t)) + B_i w(t)] \\ y(t) &= \sum_{i=1}^{p} \omega_i(x(t))(C_i x(t) + C_{di}x(t-d(t) + D_i w(t)) \\ z(t) &= \sum_{i=1}^{p} \omega_i(x(t))(E_i x(t) + E_{di}x(t-d(t)))\end{aligned} \quad (1)$$

where $x(t)$ represent the state vector of the system, $z(t)$ stands the unknown signal to be evaluated, $y(t)$ denotes output of the system, $w(t)$ represents the noise signal which belongs to arbitrary signal and satisfy $w(t) \in L_2 \in [0, \infty)$. $A_i, A_{di}, B_i, C_i, C_{di}, D_i, E_i, E_{di}$ denote known system matrices. Time delay $d(t)$ is set as a continuously differentiable function exhibiting the properties: $0 \leq d(t) < d_0, \dot{d}(t) \leq \rho$. $\omega_i(x(t))$ stands for the grade of the normalized membership satisfying: $\omega_i(x(t)) \geq 0$ and $\sum_{i=1}^{c} \omega_i(\mathbf{x}(t)) = 1$.

Filter Rule j:

Motivated by the work of [8], we plan to design the polynomial fuzzy filter of order c as follows

$$\dot{x}_f(t) = \sum_{j=1}^{c} m_j(x_f(t))(A_{fj}x_f(t) + B_{fj}y(t)) \quad (2)$$

$$z_f(t) = \sum_{j=1}^{c} m_j(x_f(t))(C_{fj}x_f(t)) \quad (3)$$

where $x_f(t)$ and $z_f(t)$ represent the state and output of the H_∞ filter respectively. And A_{fj}, B_{fj}, C_{fj} denote the filter matrices that will be designed, $j = 1, 2, \ldots, c$. $m_j(\mathbf{x}(t))$ represent the normalized membership grade exhibiting: $m_j(\mathbf{x}(t)) \geq 0$ for all i, and $\sum_{j=1}^{c} m_j(\mathbf{x}(t)) = 1$.

Based on the (1) and (2), and define the state vector of filtering error system as $\xi(t) = [x^T(t), x_f^T(t)]^T$ and $e(t) = z(t) - z_f(t)$, we can derive the whole H_∞ filtering system as follows

$$\begin{aligned}\dot{\xi}(t) = &\sum_{i=1}^{p}\sum_{j=1}^{c} \omega_i(x(t))m_j(x_f(t))(\bar{A}_i\xi(t) + \bar{A}_{di}\xi(t-d(t)) \\ &+ \bar{B}_i w(t))\end{aligned} \quad (4)$$

$$e(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} \omega_i(x(t))m_j(x_f(t))(\bar{E}_i\xi(t) + \bar{E}_{di}\xi(t-d(t))) \quad (5)$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ B_{fj}C_i & A_{fj} \end{bmatrix}, \bar{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ B_{fj}C_{di} & 0 \end{bmatrix},$$
$$\bar{B}_i = \begin{bmatrix} B_i \\ B_{fj}D_i \end{bmatrix}, \bar{E}_i = [E_i \quad -C_{fj}], \quad \bar{E}_d = [E_{di} \quad 0]$$

So far, our objective of this paper can be summarized as to derive a fuzzy filter of the form (2) satisfying the two conditions:

(1) If $w(t) = 0$, The filtering system (4) can be asymptotically stable;

(2) For the given scalar $\gamma > 0$, if $\xi(t) \equiv 0$ for $t \in [-d_0, 0]$, the following H_∞ performance can be satisfied:

$$\int_0^T \|e(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2 dt \quad (6)$$

for all the $T > 0$ and $w(t) \in L_2[0, \infty)$.

To simplify the computational complexity, we introduce the following vectors:

$$\begin{aligned}\Gamma_1 &= \begin{bmatrix}\bar{A}_i & \bar{A}_{di}^T & 0 & \bar{B}_i\end{bmatrix}, \\ \Gamma_2 &= \begin{bmatrix}\bar{E}_i(t) & \bar{E}_{di}(t) & 0 & 0\end{bmatrix}, \\ \Psi(t) &= [\xi^T(t) \quad \xi^T(t-d(t)) \quad \xi^T(t-h) \quad w(t)]^T.\end{aligned} \quad (7)$$

So we have

$$\dot{\xi}(t) = \Gamma_1\xi(t), \quad e(t) = \Gamma_2 e(t) \quad (8)$$

In addition, the following lemma are playing an important role in the discussion of the following section.

Lemma 1 [12]: For matirx $N = \begin{bmatrix} -R & L \\ L^T & -R \end{bmatrix} \leq 0, d(t) \in (0, h]$, and a vector function $\dot{x} : [-h, 0) \longrightarrow R^n$, the following inequality is true.

$$-h \int_{t-h}^{t} \dot{x}^T(t)R\dot{x}(t) \leq \eta^T(t)\mathit{\Gamma}\eta(t) \quad (9)$$

where

$$\mathit{\Gamma} = \begin{bmatrix} -R & R+L & -L \\ * & -2R-L-L^T & R+L \\ * & * & -R \end{bmatrix}$$
$$\eta(t) = \begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-h) \end{bmatrix}$$

III. Main Results

First the stability condition of (4) when $w(t) = 0$ is considered:

A. Stability Analysis

Theorem 1: If $w(t) = 0$, the filtering system (4) is asymptotically stable when the symmetric positive matrices P, R, Q, L which can guarantee the following LMIs hold exist

$$\Omega = \begin{bmatrix} \Xi & \sqrt{h}\Gamma_3^T P \\ * & -PR^{-1}P \end{bmatrix} < 0 \quad (10)$$

$$N = \begin{bmatrix} -R & L \\ * & -R \end{bmatrix} \leq 0 \quad (11)$$

where

$$\Xi = \begin{bmatrix} \Delta_1 & \Delta_2 & -\frac{L}{h} \\ * & \Delta_3 & \frac{1}{h}(R+L) \\ * & * & -\frac{1}{h}R \end{bmatrix},$$

$$\Delta_1 = \bar{A}_i^T P + P\bar{A}_i + Q - \frac{1}{h}R,$$

$$\Delta_2 = P\bar{A}_{di} + \frac{1}{h}(R+L),$$

$$\Delta_3 = -(1-\rho)Q - \frac{2}{h}R - \frac{1}{h}L - \frac{1}{h}L^T,$$

$$\Gamma_3 = \begin{bmatrix} \bar{A}_i & \bar{A}_{di}^T & 0 \end{bmatrix}.$$

proof: The Lyapunov function is chosen as:

$$\begin{aligned} V(t,\xi(t)) =& \xi^T(t)P\xi(t) + \int_{t-d(t)}^{t} \xi^T(s)Q\xi(s)ds \\ &+ \int_{-h}^{0}\int_{t+\theta}^{t} \dot{\xi}(s)^T R\dot{\xi}(s)dsd\theta \end{aligned} \tag{12}$$

Differentiating the (12) yields

$$\begin{aligned} \dot{V}(\xi(t)) =& 2\xi^T(t)P\dot{\xi}(t) + \xi^T(t)Q\xi(t) \\ &- (1-\dot{d}(t))\xi^T(t-d(t))Q\xi(t-d(t)) \\ &+ h\dot{\xi}(t)^T R\dot{\xi}(t) - \int_{t-h}^{t} \dot{\xi}(s)^T R\dot{\xi}(s)ds \end{aligned} \tag{13}$$

Based on the Lemma 1 and the property of the time-delay, we can derive

$$\begin{aligned} \dot{V}(\xi(t)) \leq& 2\xi^T(t)P\dot{\xi}(t) + \xi^T(t)Q\xi(t) - (1-\rho)\xi^T(t \\ &- d(t))Q\xi(t-d(t)) + h\dot{\xi}(t)^T R\dot{\xi}(t) + \eta^T(t)\Gamma\eta(t) \end{aligned} \tag{14}$$

note that $\sum_{i=1}^{p}\omega_i = \sum_{j=1}^{c} m_j = \sum_{i=1}^{p}\sum_{j=1}^{c}\omega_i m_j = 1$, we have

$$\dot{V}(\xi(t)) \leq \sum_{i=1}^{p}\sum_{j=1}^{c}\omega_i m_j \eta^T(t)\Omega\eta(t) \tag{15}$$

where Ω is defined as (10), and according to Lyapunov stability theory, Theorem 1 hold. Thus the proof is completed.

Next we will give a criterion that not only the asympotically stable condition can be satisfied, but also the second condition (6) will be met.

Theorem 2: The filtering system (4) can be asymptotically stable, as well as the second condition of the the H_∞ (6) can be met, when symmetric positive matrices P, R, Q, L satisfying the LMIs (16),(17) exist:

$$\Omega_1 = \begin{bmatrix} \Xi_2 & \sqrt{h}\Gamma_1^T P & \Gamma_2^T \\ * & -PR^{-1}P & 0 \\ * & * & -I \end{bmatrix} \tag{16}$$

$$N = \begin{bmatrix} -R & L \\ L^T & -R \end{bmatrix} \leq 0 \tag{17}$$

where

$$\Xi_2 = \begin{bmatrix} \Delta_1 & \Delta_2 & -\frac{L}{h} & P\bar{B}_i \\ * & \Delta_3 & \frac{1}{h}(R+L) & 0 \\ * & * & -\frac{1}{h}R & 0 \\ * & * & * & -\gamma^2 \end{bmatrix},$$

and $\Delta_1, \Delta_2, \Delta_3$ are defined in Theorem 1, and Γ_1, Γ_2 are defined in (7).

proof: Assume $\xi(t) = 0, t \in [d_0, 0]$, and the Lyapunov candidate is set as (12), applying similar derivation process, we have

$$\dot{V}(\xi(t)) = \sum_{i=1}^{p}\sum_{j=1}^{c} h_{ij}\Psi^T(t)\Omega_2\Psi(t) \tag{18}$$

where

$$\Omega_2 = \begin{bmatrix} \Xi_1 & \sqrt{h}\Gamma_1^T P \\ * & -PR^{-1}P \end{bmatrix}$$

$$\Xi_1 = \begin{bmatrix} \Delta_1 & \Delta_2 & -\frac{L}{h} & P\bar{B}_i \\ * & \Delta_3 & \frac{1}{h}(R+L) & 0 \\ * & * & -\frac{1}{h}R & 0 \\ * & * & * & 0 \end{bmatrix}$$

and Γ_1 is defined as (7).

As $V(T) \geq 0$ and $V(0) = 0$ when $t = 0$, we can derive

$$\begin{aligned} J(T) =& \int_0^T (e^T(t)e(t) - \gamma^2 w^T(t)w(t))dt \\ \leq& \int_0^T (e^T(t)e(t) - \gamma^2 w^T(t)w(t))dt + V(T) - V(0) \\ =& \int_0^T (e^T(t)e(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t))dt \\ =& \int_0^T (\sum_{i=1}^{p}\sum_{j=1}^{c}\omega_i m_j \Psi^T(t)\Omega_1\Psi(t))dt \end{aligned} \tag{19}$$

Thus we can conclude that if the LMIs (17) and (16) hold, then (6) will be true for all $T > 0$ and any nonzero $w(t) \in L_2[0,\infty)$. To obtain less conservative result, we will derive a novel criterion which is membership function information dependent in the following part.

Denote $\sum_{i=1}^{p}\sum_{j=1}^{c}\omega_i m_j = \sum_{i=1}^{p}\sum_{j=1}^{c} h_{ij}$, according to

(19) and using some algebraic manipulations, we have

$$\begin{aligned}J(T) &= \sum_{i=1}^{p}\sum_{j=1}^{c} h_{ij}\Psi^T(t)\Omega_1\Psi(t)\\ &\leq \sum_{i=1}^{p}\sum_{j=1}^{c} h_{ij}\Psi^T(t)\Omega_1\Psi(t) + \sum_{i=1}^{p}\sum_{j=1}^{c}(\bar{h}_{ij}\\ &\quad - h_{ij})\Psi^T(t)M_{ij}\Psi(t)\\ &= \sum_{i=1}^{p}\sum_{j=1}^{c} h_{ij}\Psi^T(t)(\Omega_1 - M_{ij})\Psi(t)\\ &\quad + \sum_{i=1}^{p}\sum_{j=1}^{c}\bar{h}_{ij}\Psi^T(t)M_{ij}\Psi(t)\\ &= \sum_{i=1}^{p}\sum_{j=1}^{c} h_{ij}\Psi^T(t)(\Omega_1 - M_{ij} + \sum_{r=1}^{p}\sum_{s=1}^{c}\bar{h}_{rs}M_{rs})\Phi(t)\end{aligned} \tag{20}$$

where $\bar{h}_{ij} \geq h_{ij}$ is the upper bound of the h_{ij}, $M_{ij} = M_{ij}^T \geq 0$. Then we have the following condition:

Theorem 3: The filtering system (4) can be asymptotically stable, as well as the second condition of the the H_∞ (6) can be met, when symmetric positive matrices P, R, Q, L satisfying the LMIs (21) and (22) hold

$$\Omega_1 - M_{ij} + \sum_{i=1}^{p}\sum_{j=1}^{c}\bar{h}_{ij}M_{ij} < 0 \tag{21}$$

$$N = \begin{bmatrix} -R & L \\ L^T & -R \end{bmatrix} \leq 0 \tag{22}$$

where Ω_1 is defined in (16) and $\bar{h}_{ij} \geq h_{ij}$ is the upper bound of the h_{ij}.

Remark 1: Theorem 3 presents an improved membership function information dependent stability condition for TSFMB system (4). It is less conservative, this is because the membership functions of the FMB system is included in, and also because one can freely choose membership functions of the fuzzy filter. It is the main contribution compared with PDC-based methods.

B. H_∞ Filter Design

In this part, in the light of discussions above, we will design corresponding fuzzzy H_∞ filter for the system (1).

Theorem 4: For fuzzy system (1), if the attenuation level $\gamma > 0$ and scalar σ, ρ are given, an feasible H_∞ filter (2) exists when the symmetric positive matrices $\tilde{Q}, \tilde{R}, \tilde{L}, \tilde{A}_f, \tilde{B}_f, \tilde{C}_f$ and symmetric semi-positive matrices $\tilde{M}_{ij}$ satisfying the LMIs (23)-(36) exist.

$$\tilde{P} = \begin{bmatrix} P_{11} & \tilde{P}_{22} \\ \tilde{P}_{22} & \tilde{P}_{22} \end{bmatrix} > 0 \tag{23}$$

$$\Omega_3 - M_{ij} + \sum_{i=1}^{p}\sum_{j=1}^{c}\bar{h}_{ij}M_{ij} < 0 \tag{24}$$

$$\tilde{N} = \begin{bmatrix} -\tilde{R} & \tilde{L} \\ \tilde{L}^T & -\tilde{R} \end{bmatrix} \leq 0 \tag{25}$$

where

$$\Omega_3 = \begin{bmatrix} \Delta_4 & \Delta_5 & -\frac{\tilde{L}}{h} & \tilde{\Xi}_3 & \sqrt{h}\tilde{\Xi}_1^T & \Delta_6 \\ * & \Delta_2 & \frac{\tilde{R}+\tilde{L}}{h} & 0 & \sqrt{h}\tilde{\Xi}_2^T & \bar{E}_d \\ * & * & \frac{\tilde{R}}{h} & 0 & 0 & 0 \\ * & * & * & -\gamma^2 & \sqrt{h}\tilde{\Xi}_3^T & 0 \\ * & * & * & * & -2\sigma P + \sigma^2 R & 0 \\ * & * & * & * & * & -1 \end{bmatrix},$$

$$\Delta_4 = Sym\{\tilde{\Xi}_1\} + \tilde{Q} - \frac{\tilde{R}}{h},$$

$$\Delta_5 = \tilde{\Xi}_2 + \frac{\tilde{R}+\tilde{L}}{h},$$

$$\tilde{\Xi}_1 = \begin{bmatrix} P_{11}A_i + \tilde{B}_{fj}C_i & \tilde{A}_{fj} \\ \tilde{P}_{22}A_i + \tilde{B}_{fj}C_i & \tilde{A}_{fj} \end{bmatrix},$$

$$\tilde{\Xi}_2 = \begin{bmatrix} P_{11}A_{di} + \tilde{B}_{fj}C_{di} & 0 \\ \tilde{P}_{22}A_{di} + \tilde{B}_{fj}C_{di} & 0 \end{bmatrix},$$

$$\tilde{\Xi}_3 = \begin{bmatrix} P_{11}B_i + \tilde{B}_{fj}D_i \\ \tilde{P}_{22}B_i + \tilde{B}_{fj}D_i \end{bmatrix},$$

$$\Delta_6 = \begin{bmatrix} E_i & -\tilde{C}_{fj} \end{bmatrix}.$$

And a feasible H_∞ filter realization can be obtained by

$$A_{fj} = \tilde{P}_{22}^{-1}\tilde{A}_{fj}, \quad B_{fj} = \tilde{P}_{22}^{-1}\tilde{B}_{fj}, \quad C_{fj} = \tilde{C}_{fj} \tag{26}$$

Proof: Firstly, for any scalar δ, we have $(\delta R - P)R^{-1}(\delta R - P) \geq 0$, then we have the following condition

$$-PR^{-1}P \leq -2\delta P + \delta^2 R \tag{27}$$

using the inequality (27), we can get

$$\Omega_1 \leq \Omega_4 = \begin{bmatrix} \Delta_1 & \Delta_2 & -\frac{L}{h} & P\bar{B}_i & \sqrt{h}\bar{A}_i^T P & \bar{E}^T(t) \\ * & \Delta_3 & \frac{R+L}{h} & 0 & \sqrt{h}\bar{A}_{di}^T P & \bar{E}_d^T(t) \\ * & * & -\frac{1}{h}R & 0 & 0 & 0 \\ * & * & * & -\gamma^2 & \sqrt{h}\bar{B}_i^T P & 0 \\ * & * & * & * & -\frac{-2\delta P+\delta^2 R}{h} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{28}$$

As a result, (21) holds if the following inequality holds

$$\Omega_4 - M_{ij} + \sum_{i=1}^{p}\sum_{j=1}^{c}\bar{h}_{ij}M_{ij} < 0 \tag{29}$$

Introduce the partition as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \tag{30}$$

where $P_{11} > 0, P_{22} > 0$, and it can be supposed that P_{12} is invertible via introducing a tiny perturbation if it is necessary, Let

$$\Upsilon = \begin{bmatrix} I & 0 \\ 0 & P_{12}P_{22}^{-1} \end{bmatrix} \tag{31}$$

Denote $\mathscr{T} = diag\{\Upsilon, \Upsilon, \Upsilon, I, \Upsilon, I\}$, and using $\mathscr{T}^T$ and $\mathscr{T}$ to pre- and post-multiply (29), respectively. Then we can obtain (24) with the following changes of variable

$$\tilde{P}_{22} = P_{12}P_{22}^{-1}P_{12}^T, \quad \tilde{R} = \Upsilon^T R\Upsilon, \quad \tilde{Q} = \Upsilon^T Q\Upsilon,$$
$$\tilde{L} = \Upsilon^T L\Upsilon, \quad \tilde{M}_{ij} = \mathscr{T}^T M_{ij}\mathscr{T}, \quad \tilde{A} = P_{12}A_f P_{22}^{-1}P_{12}^T,$$
$$\tilde{B} = P_{12}B_f, \quad \tilde{C} = C_f P_{22}^{-1}P_{12}^T$$

Further, the transfer function of the H_∞ can be presented as

$$T_{z_f y}(s) = C_f(t)(sI - A_f)^{-1}B_f(t) \tag{32}$$

And using the filter matrices introduced in front yields

$$T_{z_f y}(s) = \tilde{C}_f(t)(sI - \tilde{P}_{22}^{-1}\tilde{A}_f)^{-1}\tilde{P}_{22}^{-1}\tilde{B}_f(t) \tag{33}$$

So far, we complete the proof of Theorem 4.

The criterion introduced in Theorem 4 can produce satisfactory results, however, it is necessary to point out that the following Corollary also work to address fuzzy H_∞ filter design problem.

Corollary 1: For fuzzy system (1), if the attenuation level $\gamma > 0$ and scalar σ, ρ are given, an feasible H_∞ filter (2) exists when the symmetric positive matrices $\tilde{Q}, \tilde{R}, \tilde{L}, \tilde{A}_f, \tilde{B}_f, \tilde{C}_f$ satisfying the LMIs (34)-(36) hold.

$$\tilde{P} = \begin{bmatrix} P_{11} & \tilde{P}_{22} \\ \tilde{P}_{22} & \tilde{P}_{22} \end{bmatrix} > 0 \tag{34}$$

$$\Omega_3 \quad < \quad 0 \tag{35}$$

$$\tilde{N} = \begin{bmatrix} -\tilde{R} & \tilde{L} \\ \tilde{L}^T & -\tilde{R} \end{bmatrix} \quad \leq \quad 0 \tag{36}$$

where Ω_3 is defined in Theorem 4.

Remark 2: Compared with Theorem 4, Corollary 1 does not contain information about the membership functions, so it will be more realizable in practice. Besides the filter designed by Corollary 1 can satisfy most engineering requirement in practice, as a result, it is presented in this paper.

IV. Numerical Examples

Next, two numerical examples will be offered to testify the validity of the derived results.

A. Example 1

Consider the TSFMB system provided in the literature [?] with

$$\omega_1(x_1(t)) = 1 - \frac{0.5}{1 + e^{-3-x_1(t)}}, \omega_2(x_1(t)) = 1 - \omega_1(x_1(t)),$$
$$m_1(x_1(t)) = 0.7 - \frac{0.5}{1 + e^{4-x_1(t)}}, m_2(x_1(t)) = 1 - m_1(x_1(t)).$$

To persuasively illustrate the validity and superiority of the designed method, we choose several groups of d_0 and σ to find the minimum attenuation level γ_{min}, as the size of γ_{min} can reflect the conservatism of the filter design method, i.e., the smaller γ_{min} means less conservatism of the method. The computational results of γ_{min} are listed in the following tables.

Table I The minimum attenuation level γ_{min} for $d_0 = 0.5$

method	$\sigma = 0.7$	$\sigma = 1$	$\sigma = 2$	$\sigma = 5$	$\sigma = 10$	$\sigma = 20$
[13]	0.59	0.38	0.35	0.34	0.34	0.37
[14]	0.42	0.27	0.25	0.24	0.24	0.26
[15]	0.4163	0.2661	0.2452	0.2371	0.2375	0.2537
Th. 4	0.33	0.22	0.20	0.19	0.19	0.20

Table II The minimum attenuation level γ_{min} for $d_0 = 0.8$

method	$\sigma = 0.7$	$\sigma = 1$	$\sigma = 2$	$\sigma = 5$	$\sigma = 10$	$\sigma = 20$
[13]	11.98	0.83	0.38	0.35	0.37	1.01
[14]	8.54	0.59	0.27	0.25	0.26	0.70
[15]	8.487	0.5821	0.2607	0.2445	0.2530	0.4338
Th. 4	0.88	0.47	0.21	0.20	0.20	0.21

I. From Table I, Table II, it can be concluded that the designed method can produce more satisfactory results than the ones in literature [13–15]. Next will provide an example to clarify the effect of the imperfect premise matching method.

B. Example 2

Consider a TSFMB system (1) involving time-delay with

$$\mathbf{A}_1 = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix};$$
$$\mathbf{A}_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \mathbf{A}_{d2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}, \mathbf{A}_{d3} = \begin{bmatrix} -1 & 0 \\ 0.5 & 0 \end{bmatrix};$$
$$\mathbf{B}_1 = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \mathbf{B}_3 = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix};$$
$$\mathbf{C}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathbf{C}_2 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}, \mathbf{C}_3 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix};$$
$$\mathbf{C}_{d1} = \begin{bmatrix} -0.8 & 0.6 \end{bmatrix}, \mathbf{C}_{d2} = \begin{bmatrix} -0.2 & 1 \end{bmatrix}, \mathbf{C}_{d3} = \begin{bmatrix} 0 & 0.5 \end{bmatrix};$$
$$\mathbf{E}_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix}, \mathbf{E}_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}, \mathbf{E}_3 = \begin{bmatrix} -0.6 & 1 \end{bmatrix};$$
$$\mathbf{E}_{d1} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \mathbf{E}_{d2} = \begin{bmatrix} 0 & 0.2 \end{bmatrix}, \mathbf{E}_{d3} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}.$$

and the membership functions are chosen as

$$\omega_1(x_1(t)) = 1 - \frac{0.6}{1 + e^{-3-x_1(t)}}, \omega_2(x_1(t)) = \frac{0.4}{1 + e^{-3-x_1(t)}},$$
$$\omega_3(x_1(t)) = 1 - \omega_1(x_1(t)) - \omega_2(x_1(t)),$$
$$m_1(x_1(t)) = 0.7 - \frac{0.5}{1 + e^{4-x_1(t)}}, m_2(x_1(t)) = 1 - m_1(x_1(t)).$$

The scalar ρ is set as $\rho = 0.2$, with diffierent σ and d_0, we can get the following results

Table III The minimum attenuation level γ_{min} for $d_0 = 0.5$

method	$\sigma = 0.7$	$\sigma = 1$	$\sigma = 2$	$\sigma = 5$	$\sigma = 10$	$\sigma = 20$
Corollary 1	0.42	0.27	0.25	0.24	0.24	0.25
Theorem 4	0.21	0.16	0.14	0.11	0.10	0.11

Table IV The minimum attenuation level γ_{min} for $d_0 = 0.8$

method	$\sigma = 0.7$	$\sigma = 1$	$\sigma = 2$	$\sigma = 5$	$\sigma = 10$	$\sigma = 20$
Corollary 1	9.63	0.60	0.27	0.25	0.26	0.72
Theorem 4	0.88	0.26	0.15	0.13	0.11	0.12

Remark 3: From Example 2, we can clearly see that the methods proposed in this paper allow the fuzzy model and the fuzzy H_∞ filter bear totally distinct membership functions and distinct number of fuzzy rules. This design can supply greater design flexibility for the fuzzy H_∞ filter design, which means the structure complexity and implementation cost can be lowered through choosing simple membership functions. What's more, it can be seen that Theorem 4 can produce less conservative results than Corollary 1, this is because Theorem 4 is membership function dependent method while Corollary 1 is membership function independent method.

V. Conclusion

This paper studies H_∞ fuzzy filter design issue for nonlinear systems with time-varying delay. The nonlinear system is described by T-S fuzzy model, and based on the Lyapunov theory and imperfect premise matching method, some improved stability criteria and new H_∞ filter design approaches have been proposed. Moreover, some numerical examples are offered to clarify the validity and superiority of the designed approach.

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