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New Results on Stability and Stabilization Analyses for T-S Fuzzy Systems with Distributed Time-Delay under Imperfect Premise Matching

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***Abstract*—This paper investigates stability and stabilization analyses issue for T-S fuzzy-model-based systems involving distributed time-delay. The novel imperfect premise matching methodlogy is employed to derive fuzzy controller, unlike conventional PDC method, this novel technique allows the designed fuzzy controller and the T-S fuzzy model to have distinct membership functions and number of rules, as a result, great flexibility and better robustness can be achieved. Moreover, with the introduction of one novel inequality which is tighter than other existing ones, relaxed LMI-based stability and stabilization criteria are obtained in term of the Lyapunov stability theory. Lastly, two numerical examples are offered to prove the valadity of the derived results.**

***Keywords*—T-S fuzzy model, distributed time-delay, Lyapunov stability, imperfect premise matching**

I. Introduction

The T-S fuzzy model [1] is significant instrument to describe the dynamics of fuzzy systems, when designing fuzzy controllers for TSFMB system, the PDC [2] method is commonly used, which assumes that the fuzzy controller to be designed and the T-S fuzzy model enjoy the uniform membership functions and uniform number of rules. Such design allows the stability analysis to be conducted effectively, however, the PDC approach can also limit the design flexibility. To address this problem, some new alternative approaches are introduced, among them, one effective method named imperfect premise matching which was proposed by Lam in 2009 [3]. This novel design technology allows the fuzzy controller to be designed and the constructed fuzzy model possess diverse membership functions and number of rules, thus successfully resolve the problems of the PDC approach. In this paper, we are going to employ this novel imperfect premise matching method to design the fuzzy controller.

What's more, time delay frequently appeared in control systems, which can pose instability of the system and deteriorate the system performance. In recent decades, many research results dealing with the time-delay cases in control systems have been proposed [4]. To investigate the control systems involving time-delay, the stability analysis issue must be addressed first, and the Lyapunov-Krasovskii functional (LKF) method is the common choice to do with such cases, this method can effectively work out the stability analysis issue in the light of the feasibility of LMIs. However, the derived results with LKF method are conservative, to reduce the conservatism, researchers have made great efforts: the triple integral/summation terms and the free-weighting matrix [5] were added to the Lyapunov function candidate; the Wirtinger-based inequality [6], the free-matrix-based inequality [7] were derived to obtain more accurate bound of the integral terms. More recently, one new integral inequality was proposed in [8], which considers the information of the double integral of the system state, and further reduces the conservatism. In the later section, We will employ this new integral inequality to resolve the stability and stabilization analyses problem for the TSFMB system.

From the discussions above, we can see that when analyzing the stability and stabilization for the TSFMB system, the conventional PDC method will limit the flexibility designing the fuzzy controller, and the derived results with LKF method are conservative. To solve these problems, we will applying the novel imperfect premise matching method to design fuzzy controller and employing the new integral inequality to develop the stability and stabilization conditions for the TSFMB system. Besides some numerical examples will be provided to clarify the valadity of the designed results.

II. Preliminaries

A TSFMB nonlinear control system with distributed time delay is considered.

Fuzzy Model:

Construct a p-rule polynomial fuzzy model to represent the nonlinear system with time-delay.

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} \omega_i(\mathbf{x}(t))\big(\mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{1i}\mathbf{x}(t-h) + \mathbf{A}_{2i}\int_{t-h}^{t}\mathbf{x}(s)ds + \mathbf{B}_i\mathbf{u}(t)\big) \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}$ denotes the vector of the system state; $\mathbf{A}_i \in \mathbb{R}^{n\times n}, \mathbf{A}_{1i} \in \mathbb{R}^{n\times n}, \mathbf{A}_{2i} \in \mathbb{R}^{n\times n}$ and $\mathbf{B}_i \in \mathbb{R}^{n\times m}$ represent the system matrices and the system input matrices; $\mathbf{u}(t) \in \mathbb{R}^{m\times 1}$ is system input; the time delay h is a constant satisfying $h \in [h_{min}, h_{max}]$; $\omega_i(\mathbf{x}(t))$ stands for the normalized grade of membership, and satisfying: $\omega_i(\mathbf{x}(t)) \geq 0$ for all i, and $\sum_{i=1}^{p} \omega_i(\mathbf{x}(t)) = 1$.

Fuzzy Controller:

In this study, a c-rule polynomial fuzzy controller to be designed to stabilize the system (5) is considered:

$$\mathbf{u}(t) = \sum_{j=1}^{p} m_j(\mathbf{x}(t))\mathbf{K}_J\mathbf{x}(t) \tag{2}$$

where $\mathbf{K}_j \in \mathbb{R}^{m\times n}$ represents the feedback gain of jth rule; $m_j(\mathbf{x}(t))$ stands for the normalized grade of membership, and satisfying: $m_j(\mathbf{x}(t)) \geq 0$ for all i, and $\sum_{j=1}^{p} m_j(\mathbf{x}(t)) = 1$.

According to (1) and (2), the closed-loop fuzzy control system can be easily acquired:

$$\begin{aligned}\dot{\mathbf{x}}(t) =& \sum_{i=1}^{p}\sum_{j=1}^{p} \omega_i(\mathbf{x}(t))m_i(\mathbf{x}(t))\big(\mathbf{G}_{ij}\mathbf{x}(t) + \mathbf{A}_{1i}\mathbf{x}(t-h) \\ &+ \mathbf{A}_{2i}\int_{t-h}^{t}\mathbf{x}(s)ds + \mathbf{B}_i\mathbf{u}(t)\big)\end{aligned} \tag{3}$$

where $\mathbf{G}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j, i = 1,2,...,p, j = 1,2,...,p$

Remark 1: From the expression (5) and expression (2), it can be clearly seen that the T-S fuzzy model and the fuzzy controller tp be designed have distinct membership functions and number of rules, which is not allowed by the conventional PDC method. The PDC method cannot deal with the nonlinear system with uncertainty, and the specific rule will complicate the structure of the designed fuzzy controller unnecessarily in some circumstances.

To simplify complex matrices and vector representations, the following nomenclature is given:

$$\begin{aligned}&\mathbf{e_i} = [\mathbf{0}_{n\times(i-1)n} \quad \mathbf{I}_n \quad \mathbf{0}_{n\times(4-i)n}], \quad i = 1,2,3,4,\\ &\eta_1(t) = \int_{t-h}^{t}\mathbf{x}(s)ds, \quad \eta_2(t) = \int_{t-h}^{t}\int_{t-h}^{s}\mathbf{x}(u)duds,\\ &\varepsilon(t) = [\mathbf{x}^T(t) \quad \eta_1^T(t) \quad \eta_2^T(t)]^T,\\ &\zeta(t) = \left[\mathbf{x}^T(t) \quad \mathbf{x}^T(t-h) \quad \tfrac{1}{h}\eta_1^T(t) \quad \tfrac{2}{h^2}\eta_2^T(t)\right]^T,\\ &\mathbf{\Upsilon}_i = [\mathbf{A}_i \quad \mathbf{A}_{1i} \quad h\mathbf{A}_{2i} \quad \mathbf{0}].\end{aligned}$$

In addition, to simplify computational complexity, $\omega_i(\mathbf{x}(t)),\ i = 1,2,...,p$ and $m_j(\mathbf{x}(t)),\ j = 1,2,...,p$ are denoted as $\omega_i(\mathbf{x}(t)) = \omega_i$ and $m_j(\mathbf{x}(t)) = m_j$ in the following sections.

Besides, the following lemma is playing a significant role in the later section.

Lemma 1 [8]: Assuming that function $\mathbf{x}$ can be differentiated, and satisfy: $[\alpha,\beta] \rightarrow \mathbb{R}^n$. For $\mathbf{N}_1, \mathbf{N}_2, \mathbf{N}_3 \in \mathbb{R}^{4n\times n}$, and $\mathbf{R} \in \mathbb{R}^{n\times n} > 0$, the following inequality is true:

$$-\int_{\alpha}^{\beta}\dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds \leq \xi^T\Omega\xi, \tag{4}$$

where

$$\begin{aligned}\Omega =& \tau(\mathbf{N}_1\mathbf{R}^{-1}\mathbf{N}_1^T + \frac{1}{3}\mathbf{N}_2\mathbf{R}^{-1}\mathbf{N}_2^T + \frac{1}{5}\mathbf{N}_3R^{-1}\mathbf{N}_3^T)\\ &+ Sym\{\mathbf{N}_1\Delta_1 + \mathbf{N}_2\Delta_2 + \mathbf{N}_3\Delta_3\},\\ \Delta_1 =& \mathbf{e}_1 - \mathbf{e}_2, \quad \Delta_2 = \mathbf{e}_1 + \mathbf{e}_2 - 2\mathbf{e}_3,\\ \Delta_3 =& \mathbf{e}_1 - \mathbf{e}_2 - 6\mathbf{e}_3 + 6\mathbf{e}_4,\\ \xi =& [\mathbf{x}^T(\beta) \quad \mathbf{x}^T(\alpha) \quad \tfrac{1}{\tau}\textstyle\int_{\alpha}^{\beta}\mathbf{x}^T(s)ds \quad \tfrac{2}{\tau^2}\int_{\alpha}^{\beta}\int_{\alpha}^{s}\mathbf{x}^T(u)duds]^T,\\ \tau =& \beta - \alpha.\end{aligned}$$

III. Main Results

Firstly, the stability condition of the autonomous system (5) will be investigated.

A. Stability Analysis

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p}\omega_i(\mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{1i}\mathbf{x}(t-h) + \mathbf{A}_{2i}\int_{t-h}^{t}\mathbf{x}(s)ds). \tag{5}$$

Theorem 1: For the given constant $h \in [h_{min,h_{max}}]$, the TSFMB autonomous system (5) can be asymptotically stable when symmetric matrices $\mathbf{P}, \mathbf{R}, \mathbf{Q}$, and symmetric semi-positive matrices $\mathbf{F_{ij}}, \mathbf{T_{ij}}$ exist, so that the following LMIs hold.

$$\mathbf{\Phi}_i = \begin{bmatrix}\mathbf{\Phi}_{1i} + \mathbf{\Phi}_3 & \sqrt{h}\mathbf{N}_1 & \sqrt{h}\mathbf{N}_2 & \sqrt{h}\mathbf{N}_3\\ * & -\mathbf{R} & \mathbf{0} & \mathbf{0}\\ * & * & -3\mathbf{R} & \mathbf{0}\\ * & * & * & -5\mathbf{R}\end{bmatrix} < 0, \tag{6}$$

where

$$\begin{aligned}&\mathbf{\Phi}_{1i} = Sym\{\Delta_4^T\mathbf{P}\mathbf{\Delta}_{5i}\} + \mathbf{e}_1^T\mathbf{Q}\mathbf{e}_1 - \mathbf{e}_2^T\mathbf{Q}\mathbf{e}_2 + h\Upsilon_i^T\mathbf{R}\Upsilon_i,\\ &\mathbf{\Delta}_4 = \begin{bmatrix}\mathbf{e}_1^T & h\mathbf{e}_3^T & \frac{h^2}{2}\mathbf{e}_4^T\end{bmatrix}^T,\\ &\mathbf{P} = \begin{bmatrix}\mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{P}_{13}\\ * & \mathbf{P}_{22} & \mathbf{P}_{23}\\ * & * & \mathbf{P}_{33}\end{bmatrix},\\ &\mathbf{\Delta}_{5i} = \begin{bmatrix}\Upsilon_i^T & \mathbf{e}_1^T - \mathbf{e}_2^T & h\mathbf{e}_3^T - h\mathbf{e}_2^T\end{bmatrix}^T,\\ &\mathbf{\Phi}_3 = Sym\{\mathbf{N}_1\mathbf{\Delta}_1 + \mathbf{N}_2\mathbf{\Delta}_2 + \mathbf{N}_3\mathbf{\Delta}_3\},\\ &i = 1,2,...,p,\end{aligned}$$

and $\mathbf{\Delta}_1, \mathbf{\Delta}_2, \mathbf{\Delta}_3$ are defined in Lemma 1.

Proof: Choosing the following Lyapunov function candidate:

$$\begin{aligned}V(t) =& \varepsilon^T(t)\mathbf{P}\varepsilon(t) + \int_{t-h}^{t}\mathbf{x}^T(s)\mathbf{Q}\mathbf{x}(s)ds\\ &+ \int_{-h}^{0}\int_{t+\theta}^{t}\dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)dsd\theta\end{aligned} \tag{7}$$

Differentiating the above equations yields:

$$\dot{V}(t) = \sum_{i=1}^{p}\omega_i\big(\zeta^T(t)\Phi_{1i}\zeta(t) - \int_{t-h}^{t}\dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds\big) \tag{8}$$

according to Lemma 1, we can obtain:

$$
\begin{aligned}
&-\int_{t-h}^{t}\dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds \\
&\leq \zeta^T(t)\Big[h\mathbf{N}_1\mathbf{R}^{-1}\mathbf{N}_1^T+\frac{h}{3}\mathbf{N}_2\mathbf{R}^{-1}\mathbf{N}_2^T+\frac{h}{5}\mathbf{N}_3\mathbf{R}^{-1}\mathbf{N}_3^T \\
&\quad + Sym\{\mathbf{N}_1\mathbf{\Delta}_1+\mathbf{N}_2\mathbf{\Delta}_2+\mathbf{N}_3\mathbf{\Delta}_3\}\Big]\zeta(t) \\
&= \zeta^T(t)\left(\mathbf{\Phi}_2+\mathbf{\Phi}_3\right)\zeta(t)
\end{aligned} \tag{9}
$$

where

$$
\Phi_2 = h\mathbf{N}_1\mathbf{R}^{-1}\mathbf{N}_1^T+\frac{h}{3}\mathbf{N}_2\mathbf{R}^{-1}\mathbf{N}_2^T+\frac{h}{5}\mathbf{N}_3\mathbf{R}^{-1}\mathbf{N}_3^T
$$

From (14), we have:

$$
\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^{p}\omega_i\zeta^T(t)(\mathbf{\Phi}_{1i}+\mathbf{\Phi}_2+\mathbf{\Phi}_3)\zeta(t) \\
&= \sum_{i=1}^{p}\omega_i\zeta^T(t)\mathbf{\Phi}_i\zeta(t)
\end{aligned} \tag{10}
$$

where $\mathbf{\Phi}_i=\mathbf{\Phi}_{1i}+\mathbf{\Phi}_2+\mathbf{\Phi}_3, i=1,2,...,p$.

So if $\mathbf{\Phi}_i<\mathbf{0}$, $\dot{V}(\mathbf{x}(t))<\mathbf{0}$ can be derived. And applying the Schur Complement theory, we can get equation (6), i.e., the system (5) is asymptotically stable when equation (6) holds, and thus accomplish the proof of Theorem 1.

For the sake of eliminating the free matrices $\mathbf{N}_1,\mathbf{N}_2,\mathbf{N}_3$ in Theorem 1 to decrease the computational complexity, the following corollary is given:

Corollary 1: For the given constant $h\in[h_{min},h_{max}]$, the TSFMB autonomous system (5) can be asymptotically stable when symmetric positive matrices $\mathbf{P},\mathbf{R},\mathbf{Q}$, and symmetric semi-positive matrices $\mathbf{F_{ij}},\mathbf{T_{ij}}$ exist, such that $\Phi_i<0$, $i=1,2,..,p$, where Φ_i is defined as Theorem 1, and $\mathbf{N}_1,\mathbf{N}_2,\mathbf{N}_3$ satisfy the following equations

$$
\begin{aligned}
\mathbf{N}_1 &= \frac{1}{h}\begin{bmatrix}-\mathbf{R} & \mathbf{R} & \mathbf{0} & \mathbf{0}\end{bmatrix}^T, \\
\mathbf{N}_2 &= \frac{3}{h}\begin{bmatrix}-\mathbf{R} & -\mathbf{R} & 2\mathbf{R} & \mathbf{0}\end{bmatrix}^T, \\
\mathbf{N}_3 &= \frac{5}{h}\begin{bmatrix}-\mathbf{R} & \mathbf{R} & 6\mathbf{R} & -6\mathbf{R}\end{bmatrix}^T.
\end{aligned}
$$

Though the Theorem 1 and Corollary 1 can successfully address the stability analysis problem of (5), neither of them could be used to construct fuzzy controller for the aforementioned system. Next we would mainly investigate how to construct the fuzzy controller to stabilize the system (5).

B. Stabilization Analysis

Theorem 2: For the given constants $h\in[h_{min},h_{max}]$, σ and $t_i, i=2,3,..,6$, the fuzzy system (3) can be asymptotically stable when the following inequality $m_j(\mathbf{x}(t))-\rho_j\omega_j(\mathbf{x}(t))\geq 0,\ 0<\rho_j<1,\ j=1,2,...,p$ are true for all the $\mathbf{x}(t)$ and j, and symmetric positive matrices $\bar{\mathbf{R}},\bar{\mathbf{Q}}$, $\mathbf{F}_{ij}$, $\mathbf{\Lambda}_i$, $\bar{\mathbf{K}}_j$, $\mathbf{X}$, $i=1,2,...,p$, $j=1,2,...,p$ exist, so that the LMIs (24), (27)-(29) hold. And under such cirmustance, the state feedback gain can be represented as: $\mathbf{K}_j=\bar{\mathbf{K}}_j\mathbf{X}^{-1}$.

Proof: Just like the proof process of Theorem 1, we substitute $\mathbf{A}_i$ for $\mathbf{G}_{ij}=\mathbf{A}_i+\mathbf{B}_i\mathbf{K}_j$, and since the TSFMB control system with distributed time-delay is (3) now, equation (8) will be instead by the following equation:

$$
\dot{V}(t)=\sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i m_j\big(\zeta^T(t)\mathbf{\Phi}_{1ij}\zeta(t)-\int_{t-h}^{t}\dot{x}^T(s)\mathbf{R}\dot{x}(s)ds\big) \tag{11}
$$

and further, we have:

$$
\dot{V}(t)\leq\sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i m_j\zeta^T(t)(\mathbf{\Phi}_{1ij}+\mathbf{\Phi}_2+\mathbf{\Phi}_3)\zeta(t) \tag{12}
$$

The $\zeta^T(t)(\mathbf{\Phi}_{1ij}+\mathbf{\Phi}_2+\mathbf{\Phi}_3)\zeta(t)$ can also be denoted as $\zeta^T(t)(\mathbf{M}_{1i}+\mathbf{M}_2+\mathbf{M}_{3ij}+\sum_{i=4}^{9}\mathbf{M}_i\zeta(t)$.
where

$$
\mathbf{M}_{1ij}=
\begin{bmatrix}
\mathbf{P}_{11}\mathbf{G}_{ij}+\mathbf{P}_{12} & \mathbf{\Pi}_2 & h\mathbf{P}_{11}\mathbf{A}_{2i}+\mathbf{P}_{13}h & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
h\mathbf{P}_{12}^T\mathbf{G}_{ij}+h\mathbf{P}_{22} & \mathbf{\Pi}_3 & h^2\mathbf{P}_{12}^T+h^2\mathbf{P}_{23} & \mathbf{0} \\
\frac{h^2}{2}\mathbf{P}_{13}^T\mathbf{G}_{ij}+\frac{h^2}{2}\mathbf{P}_{23} & \mathbf{\Pi}_4 & \frac{h^2}{2}\mathbf{P}_{13}^T\mathbf{A}_{2i}+\frac{h^2}{2}\mathbf{P}_{33} & \mathbf{0}
\end{bmatrix},
$$

$$
\mathbf{\Pi}_2=\mathbf{P}_{11}\mathbf{A}_{1i}-\mathbf{P}_{12}-h\mathbf{P}_{13},
$$

$$
\mathbf{\Pi}_3=h\mathbf{P}_{12}^T\mathbf{A}_{1i}-h\mathbf{P}_{22}-h^2\mathbf{P}_{23},
$$

$$
\mathbf{\Pi}_4=\frac{h^2}{2}\mathbf{P}_{13}^T\mathbf{A}_{1i}-\frac{h^2}{2}\mathbf{P}_{23}^T-\frac{h^3}{2}\mathbf{P}_{33},
$$

$$
\mathbf{M}_2=\begin{bmatrix}\mathbf{Q} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{Q} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},
$$

$$
\mathbf{M}_{3ij}=
\begin{bmatrix}
\mathbf{G}_{ij}^T\mathbf{R}\mathbf{G}_{ij} & \mathbf{G}_{ij}^T\mathbf{R}\mathbf{A}_{1i} & \mathbf{G}_{ij}^T\mathbf{R}(h\mathbf{A}_{2i}) & \mathbf{0}\\
\mathbf{A}_{1i}^T\mathbf{R}\mathbf{G}_{ij} & \mathbf{A}_{1i}^T\mathbf{R}\mathbf{A}_{1i} & \mathbf{A}_{1i}^T\mathbf{R}(h\mathbf{A}_{2i}) & \mathbf{0}\\
h\mathbf{A}_{2i}^T\mathbf{R}\mathbf{G}_{ij} & h\mathbf{A}_2^T\mathbf{R}\mathbf{A}_{1i} & (h\mathbf{A}_2^T)\mathbf{R}(h\mathbf{A}_{2i}) & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{bmatrix}.
$$

$i=1,2,...,p, j=1,2,...,p,$

$$
\mathbf{M}_4=\frac{1}{h}\begin{bmatrix}\mathbf{R}^T & -\mathbf{R}^T & \mathbf{0} & \mathbf{0}\\ -\mathbf{R}^T & \mathbf{R}^T & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},
$$

$$
\mathbf{M}_5=\frac{3}{h}\begin{bmatrix}\mathbf{R}^T & \mathbf{R}^T & -2\mathbf{R}^T & \mathbf{0}\\ \mathbf{R}^T & \mathbf{R}^T & -2\mathbf{R}^T & \mathbf{0}\\ -2\mathbf{R}^T & -2\mathbf{R}^T & 4\mathbf{R}^T & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},
$$

$$
\mathbf{M}_6=\frac{5}{h}\begin{bmatrix}\mathbf{R}^T & -\mathbf{R}^T & -6\mathbf{R}^T & 6\mathbf{R}^T\\ -\mathbf{R}^T & \mathbf{R}^T & 6\mathbf{R}^T & -6\mathbf{R}^T\\ -6\mathbf{R}^T & 6\mathbf{R}^T & 36\mathbf{R}^T & -36\mathbf{R}^T\\ 6\mathbf{R}^T & -6\mathbf{R}^T & -36\mathbf{R}^T & 36\mathbf{R}^T\end{bmatrix}.
$$

$$
\mathbf{M}_7=Sym\left\{\frac{1}{h}\begin{bmatrix}-\mathbf{R}^T & \mathbf{R}^T & \mathbf{0} & \mathbf{0}\\ \mathbf{R}^T & -\mathbf{R}^T & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\right\},
$$

$$\mathbf{M}_8 = Sym\left\{\frac{3}{h}\begin{bmatrix} -\mathbf{R}^T & -\mathbf{R}^T & 2\mathbf{R}^T & \mathbf{0} \\ -\mathbf{R}^T & -\mathbf{R}^T & 2\mathbf{R}^T & \mathbf{0} \\ 2\mathbf{R}^T & 2\mathbf{R}^T & -4\mathbf{R}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}\right\},$$

$$\mathbf{M}_9 = Sym\left\{\frac{5}{h}\begin{bmatrix} -\mathbf{R}^T & \mathbf{R}^T & 6\mathbf{R}^T & -6\mathbf{R}^T \\ \mathbf{R}^T & -\mathbf{R}^T & -6\mathbf{R}^T & 6\mathbf{R}^T \\ 6\mathbf{R}^T & -6\mathbf{R}^T & -36\mathbf{R}^T & 36\mathbf{R}^T \\ -6\mathbf{R}^T & 6\mathbf{R}^T & 36\mathbf{R}^T & -36\mathbf{R}^T \end{bmatrix}\right\}.$$

Which means $\dot{V}(t) < 0$, if

$$\mathbf{M}_{1i} + \mathbf{M}_2 + \mathbf{M}_{3ij} + \mathbf{M}_4 + \mathbf{M}_5 + \mathbf{M}_6 + \mathbf{M}_7 + \mathbf{M}_8 + \mathbf{M}_9 < \mathbf{0} \tag{13}$$

Define some new variables :

$$\begin{aligned} &\mathbf{P}_{12} = t_2\mathbf{P}_{11}, \mathbf{P}_{13} = t_3\mathbf{P}_{11}, \mathbf{P}_{22} = t_4\mathbf{P}_{11}, \\ &\mathbf{P}_{23} = t_5\mathbf{P}_{11}, \mathbf{P}_{33} = t_6\mathbf{P}_{11}, \end{aligned} \tag{14}$$

let equation (13) be pre-multiplied and post-multiplied by $diag\begin{bmatrix}\mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X}\end{bmatrix}$ and its transpose respectively, and introduce some new variables as:

$$\begin{aligned} &\mathbf{X} = \mathbf{P}_{11}^{-1}, \bar{\mathbf{R}} = \mathbf{X}\mathbf{R}\mathbf{X}, \bar{\mathbf{Q}} = \mathbf{X}\mathbf{Q}\mathbf{X}, \\ &\bar{\mathbf{K}}_j = \mathbf{K}_j\mathbf{X}, \bar{\mathbf{G}}_{ij} = \mathbf{A}_i\mathbf{X} + \mathbf{B}_i\bar{\mathbf{K}}_j, \\ &\bar{\mathbf{A}}_{1i} = \mathbf{A}_{1i}\mathbf{X}, \bar{\mathbf{A}}_{2i} = \mathbf{A}_{2i}\mathbf{X}, \bar{\mathbf{B}}_i = \mathbf{B}_i\mathbf{X}, \\ &\bar{\mathbf{N}}_1 = \frac{1}{h}\begin{bmatrix}-\bar{\mathbf{R}} & \bar{\mathbf{R}} & \mathbf{0} & \mathbf{0}\end{bmatrix}^T, \\ &\bar{\mathbf{N}}_2 = \frac{3}{h}\begin{bmatrix}-\bar{\mathbf{R}} & -\bar{\mathbf{R}} & 2\bar{\mathbf{R}} & \mathbf{0}\end{bmatrix}^T, \\ &\bar{\mathbf{N}}_3 = \frac{5}{h}\begin{bmatrix}-\bar{\mathbf{R}} & \bar{\mathbf{R}} & 6\bar{\mathbf{R}} & -6\bar{\mathbf{R}}\end{bmatrix}^T, \\ &\bar{\mathbf{\Upsilon}}_{ij} = [\bar{\mathbf{G}}_{ij} \quad \bar{\mathbf{A}}_{1i} \quad h\bar{\mathbf{A}}_{2i} \quad \mathbf{0}], \\ &\bar{\mathbf{\Delta}}_{5ij} = \begin{bmatrix}\bar{\mathbf{\Upsilon}}_{ij}^T & \mathbf{X}(\mathbf{e}_1^T - \mathbf{e}_2^T) & h\mathbf{X}(\mathbf{e}_3^T - h\mathbf{e}_2^T)\end{bmatrix}^T, \end{aligned} \tag{15}$$

then we can obtain:

$$\begin{aligned} &Sym\{\mathbf{\Delta}_4^T\bar{\mathbf{P}}\bar{\mathbf{\Delta}}_{5ij}\} + \mathbf{e}_1^T\bar{\mathbf{Q}}\mathbf{e}_1 - \mathbf{e}_2^T\bar{\mathbf{Q}}\mathbf{e}_2 \\ &+ h\bar{\mathbf{N}}_1\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_1^T + \frac{h}{3}\bar{\mathbf{N}}_2\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_2^T + \frac{h}{5}\bar{\mathbf{N}}_3\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_3^T \\ &+ Sym\{\bar{\mathbf{N}}_1\mathbf{\Delta}_1 + \bar{\mathbf{N}}_2\mathbf{\Delta}_2 + \bar{\mathbf{N}}_3\mathbf{\Delta}_3\} + h\mathbf{\Upsilon}_{ij}^T\mathbf{R}\mathbf{\Upsilon}_{ij} \\ &< \mathbf{0} \end{aligned} \tag{16}$$

So we have:

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^{p}\sum_{j=1}^{p}\zeta^T(t)\omega_i m_j\bar{\mathbf{\Phi}}_{ij}\zeta(t) \\ &= \sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i m_j\zeta^T(t)\left(\bar{\mathbf{\Phi}}_{1ij} + \bar{\mathbf{\Phi}}_2 + \bar{\mathbf{\Phi}}_3\right) < \mathbf{0} \end{aligned} \tag{17}$$

where

$$\begin{aligned} &\bar{\mathbf{\Phi}}_{1ij} = Sym\{\Delta_4^T\bar{\mathbf{P}}\bar{\Delta}_{5ij}\} + \mathbf{e}_1^T\bar{\mathbf{Q}}\mathbf{e}_1 - \mathbf{e}_2^T\bar{\mathbf{Q}}\mathbf{e}_2 \\ &\bar{\mathbf{\Phi}}_2 = h\bar{\mathbf{N}}_1\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_1^T + \frac{h}{3}\bar{\mathbf{N}}_2\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_2^T + \frac{h}{5}\bar{\mathbf{N}}_3\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_3^T + h\mathbf{\Upsilon}_{ij}^T\mathbf{R}\mathbf{\Upsilon}_{ij} \\ &\bar{\mathbf{\Phi}}_3 = Sym\{\bar{\mathbf{N}}_1\mathbf{\Delta}_1 + \bar{\mathbf{N}}_2\mathbf{\Delta}_2 + \bar{\mathbf{N}}_3\mathbf{\Delta}_3\} \\ &i = 1, 2, ..., p, j = 1, 2, ..., p, \end{aligned}$$

Since

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i(\omega_j - m_j)\mathbf{V}_i = \sum_{i=1}^{p}\omega_i\left(\sum_{j=1}^{p}\omega_j - \sum_{j=1}^{p}m_j\right)\mathbf{V}_i = \mathbf{0} \tag{18}$$

where $\mathbf{V}_i > \mathbf{0}, i = 1, 2, ..., p$, and are arbitrary matrices. Then introduce some new terms to equation (17) to reduce conservatism:

$$\begin{aligned} \dot{V}(t &\leq \sum_{i=1}^{p}\sum_{j=1}^{p}\zeta^T(t)\omega_i m_j\bar{\mathbf{\Phi}}_{ij}\zeta(t) \\ &= \sum_{i=1}^{p}\sum_{j=1}^{p}\zeta^T(t)\omega_i m_j\bar{\mathbf{\Phi}}_{ij}\zeta(t) + \sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i(\omega_i - m_j + \rho_j\omega_j \\ &\quad - \rho_j\omega_j)\zeta^T(t)\mathbf{\Lambda}_i\zeta(t) + \sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i(\omega_j - \rho_j\omega_j)\zeta^T(t)\mathbf{\Lambda}_i\zeta(t) \\ &= \sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i\omega_j\zeta^T(t)[\rho_j\mathbf{\Phi}_{ij} + \mathbf{\Lambda}_i - \rho_j\mathbf{\Lambda}_i]\zeta(t) \\ &\quad + \sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i(m_j - \rho_j\omega_j)\zeta(t)(\mathbf{\Phi}_{ij} - \mathbf{\Lambda}_i)\zeta(t) \end{aligned} \tag{19}$$

where $\mathbf{\Lambda}_i = \mathbf{\Lambda}_i^T \in \mathbb{R}^{4n \times 4n} > \mathbf{0}, i = 1, 2, ..., p$ are arbitrary matrices, and scalar $0 < \rho_j < 1,\ j = 1, 2, ..., p$ can be chosen to satisfy $m_j - \rho_j\omega_j \geq 0$ for all the j and $\mathbf{x}(t)$.
Assuming

$$\mathbf{\Phi}_{ij} - \mathbf{\Lambda}_i < \mathbf{0} \tag{20}$$

we can obtain

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i\omega_j\zeta^T(t)[\rho_j\mathbf{\Phi}_{ij} + \mathbf{\Lambda}_i - \rho_j\mathbf{\Lambda}_i]\zeta(t) \\ &= \sum_{i}^{p}\omega_i^2\zeta^T(t)[\rho_i\mathbf{\Phi}_{ii} + \mathbf{\Lambda}_i - \rho_i\mathbf{\Lambda}_i]\zeta(t) \\ &\quad + \sum_{j=1}^{p}\sum_{i<j}\omega_i\omega_j\zeta^T(t)\big[(\rho_j\mathbf{\Phi}_{ij} + \mathbf{\Lambda}_i - \rho_j\mathbf{\Lambda}_i) \\ &\quad + (\rho_i\mathbf{\Phi}_{ji} + \mathbf{\Lambda}_j - \rho_i\mathbf{\Lambda}_j)\big]\zeta(t) \end{aligned} \tag{21}$$

So if

$$\mathbf{F}_{ii} > \rho_i\mathbf{\Phi}_{ii} + \mathbf{\Lambda}_i - \rho_i\mathbf{\Lambda}_i, \quad i = 1, 2, ..., p \tag{22}$$

$$\begin{aligned} \mathbf{F}_{ij} + \mathbf{F}_{ji} >&(\rho_j\mathbf{\Phi}_{ij} + \mathbf{\Lambda}_i - \rho_j\mathbf{\Lambda}_i) + (\rho_i\mathbf{\Phi}_{ji} + \mathbf{\Lambda}_j \\ &- \rho_i\mathbf{\Lambda}_j), \quad j = 1, 2, ..., p;\ i < j \end{aligned} \tag{23}$$

where $\mathbf{F}_{ij} = \mathbf{F}_{ij}^T,\ i, j = 1, 2, ..., p$

From equation (22) – (23), we can derive

$$\dot{V}(t) \leq \begin{bmatrix}\omega_1\zeta(t) \\ \omega_2\zeta(t) \\ \vdots \\ \omega_n\zeta(t)\end{bmatrix}^T \mathbf{F} \begin{bmatrix}\omega_1\zeta(t) \\ \omega_2\zeta(t) \\ \vdots \\ \omega_n\zeta(t)\end{bmatrix} \tag{24}$$

Consequently, if $\mathbf{F} < \mathbf{0}$ and the conditions (20), (22), (23) are satisfied, the $\dot{V}(t) < 0$, which means the fuzzy system (3) is asymptotically stable.

In addition, as $\mathbf{R} = \mathbf{R}^T > \mathbf{0}$, for any scalar σ, the following inequality holds:

$$\left(\sigma\mathbf{R}^{-1} - \mathbf{X}\right)\mathbf{R}\left(\sigma\mathbf{R}^{-1} - \mathbf{X}\right) > \mathbf{0} \tag{25}$$

then we have:

$$-\mathbf{R}^{-1} < -\frac{2}{\sigma}\mathbf{X} + \frac{1}{\sigma^2}\bar{\mathbf{R}} \tag{26}$$

Applying the Schur Complement lemma and the inequality (26) to the inequalities (20), (22), (23), we can get

$$\begin{bmatrix} \mathbf{\Pi}_1 & \bar{\mathbf{N}}_1 & \bar{\mathbf{N}}_2 & \bar{\mathbf{N}}_3 & \bar{\Upsilon}_{ij}^T \\ * & -\frac{\bar{\mathbf{R}}}{h} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\frac{3\bar{\mathbf{R}}}{h} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\frac{5\bar{\mathbf{R}}}{h} & \mathbf{0} \\ * & * & * & * & \frac{\frac{1}{\sigma^2}\bar{\mathbf{R}} - \frac{2}{\sigma}\mathbf{X}}{h} \end{bmatrix} < 0, \tag{27}$$

where $\mathbf{\Pi}_1 = \bar{\mathbf{\Phi}}_{1ij} + \bar{\mathbf{\Phi}}_3 - \mathbf{\Lambda}_i, i = 1, 2, ..., p, j = 1, 2, ..., p,$

$$\begin{bmatrix} \mathbf{\Pi}_2 & \bar{\mathbf{N}}_1 & \bar{\mathbf{N}}_2 & \bar{\mathbf{N}}_3 & \bar{\Upsilon}_{ij}^T \\ * & -\frac{\bar{\mathbf{R}}}{h\rho_i} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\frac{3\bar{\mathbf{R}}}{h\rho_i} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\frac{5\bar{\mathbf{R}}}{h\rho_i} & \mathbf{0} \\ * & * & * & * & \frac{\frac{1}{\sigma^2}\bar{\mathbf{R}} - \frac{2}{\sigma}\mathbf{X}}{h\rho_i} \end{bmatrix} < 0, \tag{28}$$

where $\mathbf{\Pi}_2 = \rho_i\bar{\mathbf{\Phi}}_{1ii} + \rho_i\bar{\mathbf{\Phi}}_3 + \mathbf{\Lambda}_i - \rho_i\mathbf{\Lambda}_i - \mathbf{Q}_{ii}, i = 1, 2, ..., p,$

$$\begin{bmatrix} \mathbf{\Pi}_3 & \bar{\mathbf{N}}_1 & \bar{\mathbf{N}}_2 & \bar{\mathbf{N}}_3 & \bar{\Upsilon}_{ij}^T \\ * & -\frac{\bar{\mathbf{R}}}{h(\rho_i+\rho_j)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\frac{3\bar{\mathbf{R}}}{h(\rho_i+\rho_j)} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\frac{5\bar{\mathbf{R}}}{h(\rho_i+\rho_j)} & \mathbf{0} \\ * & * & * & * & \frac{\frac{1}{\sigma^2}\bar{\mathbf{R}} - \frac{2}{\sigma}\mathbf{X}}{h(\rho_i+\rho_j)} \end{bmatrix} < 0, \tag{29}$$

where $\mathbf{\Pi}_3 = (\rho_j\bar{\mathbf{\Phi}}_{1ij} + \rho_j\bar{\mathbf{\Phi}}_3 + \mathbf{\Lambda}_i - \rho_j\mathbf{\Lambda}_i) + (\rho_i\bar{\mathbf{\Phi}}_{1ji} + \rho_i\bar{\mathbf{\Phi}}_3 + \mathbf{\Lambda}_j - \rho_i\mathbf{\Lambda}_j) - \mathbf{Q}_{ij} - \mathbf{Q}_{ji}, i = 1, 2, ..., p, j = 1, 2, ..., p$, thus complete the proof of Theorem 2.

IV. Numerical examples

In this section, two examples are offered to prove the validity of the derived results.

A. Example 1

Consider the TSFMB autonomous system (5) with where

$$\mathbf{A}_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \mathbf{A}_{11} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$\mathbf{A}_{12} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1.5 \end{bmatrix}, \mathbf{A}_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{A}_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\omega_1(x_1(t)) = 1 - \frac{0.5}{1 + e^{-3-x_1(t)}}, \quad \omega_2(x_1(t)) = \frac{0.5}{1 + e^{-3-x_1(t)}},$$

The computational results are listed in the following table. where "—" denotes that the maximum allowable time-delay does not exist.

TABLE I The Maximum Allowable Time-Delay for Example 1

literature	maximum time-delay
[4]	—
[9]	0.6547
[10]	1.0
[11]	1.3546
Theorem 1	1.733

Remark 3: From this example, it can be evidently seen that the stability consition developed in this paper can yield more satisfactory results than the ones in other literature, this mainly because the tighter inequality introduced in the Lemma 1 is employed to derive the stability conditions.

B. Example 2

Consider the TSFMB control system (3) with

$$\mathbf{A}_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \mathbf{A}_{11} = \begin{bmatrix} -a & 0 \\ -1 & -1 \end{bmatrix},$$
$$\mathbf{A}_{12} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1.5 \end{bmatrix}, \mathbf{A}_{21} = \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}, \mathbf{A}_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\mathbf{B}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T, \mathbf{B}_2 = \begin{bmatrix} 8 & -10 + b \end{bmatrix}^T,$$
$$\omega_1(x_1(t)) = 1 - \frac{0.5}{1 + e^{-3-x_1(t)}}, \quad \omega_2(x_1(t)) = 1 - \omega_1(x_1(t)),$$
$$m_1(x_1(t)) = 0.7 - \frac{0.5}{1 + e^{4-x_1(t)}}, \quad m_2(x_1(t)) = 1 - m_1(x_1(t)),$$

where a and b denote constant system parameters.

As Theorem 2 requires:

$$m_j(x_1(t)) - \rho_j\omega_j(x_1(t)) \geq 0, \; 0 < \rho_j < 1, \; j = 1, 2 \tag{30}$$

for all the $\mathbf{x}(t)$ and j. By means of Matlab, we can derive the feasible ranges of ρ_1 and ρ_2 are:

$$\rho_1 \leq 0.402, \; \rho_2 \leq 0.647$$

And in this example, we choose $\rho_1 = 0.4$, and $\rho_2 = 0.6$.

What's more, the following table list the maximum allowable time-delay obtained through the conditions presented in Theorem 2 with different fixed a, b, σ.

TABLE II The Maximum Allowable Time-delay With $t_2 = t_3 = \cdots = t_6 = 2$

samples of a, b, σ	Theorem 2
$a = 1, b = 5, \sigma = 1$	0.46
$a = -10, b = 5, \sigma = 1$	0.05
$a = 5, b = 5, \sigma = 1$	0.26
$a = 1, b = 5, \sigma = 2$	0.68
$a = 1, b = 5, \sigma = 5$	0.42
$a = 1, b = -10, \sigma = 1$	0.46
$a = 1, b = 10, \sigma = 1$	0.48

Further, setting $a = 1, \; b = 5, \; \sigma = 1, \; h = 0.25 \;, t_2 = t_3 = \cdots = t_6 = 2$, using the method of Theorem 2, we can get state feedback gains as: $\mathbf{K}_1 = \begin{bmatrix} -0.4955 & 0.2811 \end{bmatrix}$, $\mathbf{K}_2 = \begin{bmatrix} -0.4694 & 0.2518 \end{bmatrix}$. And the initial value of the system state is chosen as $\mathbf{x}(t) = \begin{bmatrix} 3 & -2 \end{bmatrix}^T$. By means of Matlab, we can obtain Fig 1, which displays the state response of the

TSFMB control system. The trend of the changes of Fig 1 can prove that the designed fuzzy controller can achieve the function of stabilizing the system.

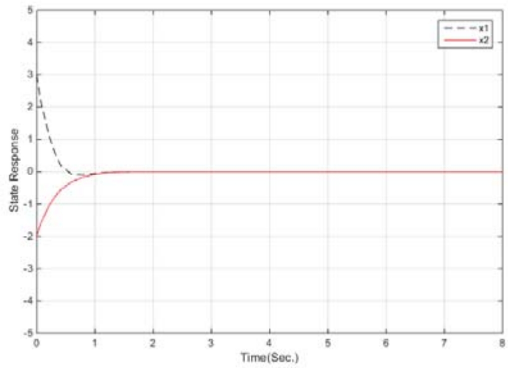

Fig. 1. State response of the TSFMB control system

V. Conclusion

The paper concerns stability and stabilization analyses problem of TSFMB system with distributed time-delay. In the light of Lyapunov theory and one novel integral inequality proposed by the literature [8], relaxed stability and stabilization conditions have been derived. Moreover, the imperfect premise matching method is employed in the stabilization analysis, consequently, more design flexibility and better robustness can be achieved. Finally, some numerical examples have been presented to further clarify the edge of the designed conditions.

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