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A New Fuzzy H_∞ Filter Design for Nonlinear Time-Delay Systems with Mismatched Premise Membership Functions *

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Abstract: This paper is concerned with the fuzzy H_∞ filter design issue for nonlinear systems with time-varying delay. To overcome the limitations of conventional methods with matched preconditions, the filter to be designed and the T-S fuzzy model are assumed to have different premise membership functions and different number of rules. Hence, greater design flexibility can be obtained. Moreover, in order to reduce conservatism, a novel integral inequality which is tighter than the traditional inequalities derived from the Leibniz-Newton formula is applied. Furthermore, the proposed filter design approach also considered the information about the membership functions, which can help further relax the derived results. Finally, two numerical examples are provided to demonstrate the effectiveness and superiority of the proposed method.

Keywords: Fuzzy H_∞ filter, Mismatched conditions, New integral inequality, Time Delay, T-S fuzzy model

1. INTRODUCTION

Time-delay is a common phenomenon in practical control systems, which can deteriorate the system performance and even cause instability. Therefore, the study for systems with time-delay is meaningful, and researchers have devoted great efforts to investigate them (Cao and Frank [2000], Wu et al. [2004], Su et al. [2009]) in last decades. When dealing with analysis and synthesis issues for nonlinear systems with time-delay, the T-S (Takagi-Sugeno) fuzzy model is often employed, which can systematically represent a nonlinear plant as a weighted sum of some linear models. In this paper, we will mainly investigate T-S fuzzy-model-based systems with time-varying delay. Moreover, filtering technology is playing a crucial role in signal processing. After several decades' development, fruitful research results about filter design have been obtained (Gao and Wang [2003], Huang et al. [2011], Qiu et al. [2009]). Among them, H_∞ filter (Tseng and Chen [2001]) has attracted widespread attention (Lin et al. [2008], Su et al. [2009], Zhang et al. [2009]), as H_∞ filter has no particular requirement for external noise signal and it is not sensitive to uncertainty.

For fuzzy H_∞ filter design issue, various analytical approaches can be found in literature. To mention a few, in (Lin et al. [2008]), a delay-dependent fuzzy H_∞ filter design method was proposed for T-S fuzzy-model-based system with time-varying delay. However, in this paper, the Lyapunov-Krasovskii function candidate was chosen as a single Lyapunov function. To obtain more relaxed results, the literature (Zhang et al. [2009]) used a fuzzy Lypunov function to analyze the stability condition. In (Su et al. [2009]), the fuzzy H_∞ filter design approach was improved by estimating the upper bound of the derivative of Lyapunov function without ignoring any useful terms. On the basis of (Su et al. [2009]), literature (Huang et al. [2011]) proposed a technique to obtain more accurate upper bound of the derivative of Laypunov function.

However, all the mentioned design methods require matching conditions, i.e, the fuzzy filter and the fuzzy model are assumed to have the same membership functions. Such assumption can facilitate the design process, but on the other hand, it will also limit the design flexibility. To resolve this problem, in this paper, we allow the fuzzy H_∞ filter to be designed and the T-S fuzzy model to have different premise membership functions and different number of rules, which is motivated by the imperfect premise matching fuzzy controller design method (Lam and Narimani [2009]). As a result, the limitation of the traditional

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fuzzy H_∞ design method with matching preconditions can be avoided. Nevertheless, such design will also make the designed filter tend to be more conservative, which means we have to find some techniques to reduce conservatism.

When analyzing stability conditions for the filtering error system, the inequalities derived from the Leibniz-Newton formula is often employed to deal with the integral term $\int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s)ds$ in the derivative of the Lyapunov function, just like in literature (Huang et al. [2011], Lin et al. [2008], He et al. [2007], Qiu et al. [2009], Lin et al. [2007]). Though such methods can work in stability analysis, the derived results are conservative, and there is little room left to further reduce conservatism. Therefore, in this paper, we will use a novel tighter integral inequality to substitute the conventional inequalities derived from the Leibniz-Newton formula to analyze stability conditions, which can help relax the derived results.

What's more, most of the existing fuzzy H_∞ filter design methods are membership functions independent (Huang et al. [2011], Lin et al. [2008], He et al. [2007]). To the best of our knowledge, membership functions dependent fuzzy H_∞ filter design approach is yet to be fully investigated. Therefore, to further reduce the conservatism, we will use a technique to introduce the information of the membership functions in the criterion.

To obtain a fuzzy H_∞ filter design method with greater design flexibility, we will allow the T-S fuzzy model and the fuzzy filter to be designed to have different premise membership functions and number of fuzzy rules. Besides, to relax the results, a novel integral inequality which is tighter than the conventional inequalities will be applied, and the information of the membership functions will be taken into account as well.

2. PRELIMINARIES

T-S Fuzzy Model: Construct a p-rule polynomial fuzzy model to represent the nonlinear system with time-delay:

$$\dot{x}(t) = \sum_{i=1}^{p} \phi_i(\psi(t))[A_i x(t) + A_{\tau i}x(t-\tau(t)) + B_i w(t)],$$
$$y(t) = \sum_{i=1}^{p} \phi_i(\psi(t))(C_i x(t) + C_{\tau i}x(t-\tau(t) + D_i w(t)),$$
$$z(t) = \sum_{i=1}^{p} \phi_i(\psi(t))(E_i x(t) + E_{\tau i}x(t-\tau(t))). \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system states, $z(t) \in \mathbb{R}^q$ is the unknown signal to be estimated, $y(t) \in \mathbb{R}^m$ is the system output, $w(t) \in \mathbb{R}^p$ is the noise signal which is assumed to be arbitrary and belongs to $L_2 \in [0,\infty)$; $A_i, A_{\tau i}, B_i, C_i, C_{\tau i}, D_i, E_i, E_{\tau i}$ are given system matrices; Time delay $\tau(t)$ is a continuously differentiable function, satisfying the conditions:

$$0 \le \tau(t) < h, \qquad \dot{\tau}(t) \le \rho. \qquad (2)$$

And $\phi_i(\psi(t)) = \Pi_{\alpha=1}^m \mu_{\mathcal{M}_\alpha^i}(\psi_\alpha(t)) / \sum_{k=1}^p \Pi_{\alpha=1}^m \mu_{\mathcal{M}_\alpha^k}(\psi_\alpha(t))$ is the normalized membership function satisfying: $\phi_i(\psi(t)) \ge 0, \sum_{i=1}^p \phi_i(\psi(t)) = 1$; $\mu_{\mathcal{M}_\alpha^i}(\psi_\alpha(t))$ is the grade of membership function which corresponds to the fuzzy term $\mathcal{M}_\alpha^i$.

To eliminate the limitations of the conventional fuzzy H_∞ filter design methods with matching preconditions. In this paper, we allow the fuzzy H_∞ filter to freely choose the premise membership functions and number of fuzzy rules, i.e., the membership functions of the T-S fuzzy model and the fuzzy filter are allowed to be different. Therefore, we assume the number of the fuzzy filter is c, and it can be represented as:

$$\dot{x_f}(t) = \sum_{j=1}^{c} n_j(g(t))(A_{fj}x_f(t) + B_{fj}y(t)),$$
$$z_f(t) = \sum_{j=1}^{c} n_j(g(t))C_{fj}x_f(t), \qquad (3)$$

where $x_f(t) \in \mathbb{R}^n$ and $z_f(t) \in \mathbb{R}^q$ are the state and output of the fuzzy H_∞ filter respectively. And A_{fj}, B_{fj}, C_{fj} are the filter matrices of that will be designed. And $n_j(g(t)) = \Pi_{\beta=1}^\Theta \mu_{\mathcal{N}_\beta^j}(g_\beta(x_f(t))) / \sum_{k=1}^c \Pi_{\beta=1}^\Theta \mu_{\mathcal{N}_\beta^k}(g_\beta(x_f(t)))$ is the normalized membership function satisfying: $\sum_{j=1}^c n_j(g(t)) = 1, n_j(g(t)) \geqslant 0$. $\mu_{\mathcal{N}_\beta^j}(g_\beta(x_f(t)))(\ \beta = 1,2,\ldots,\Theta)$ is the grade of membership functions which corresponds to the fuzzy term $\mathcal{N}_\beta^j$.

According to (1) and (3), and define the augmented state vector as $\zeta(t) = [x^T(t), x_f^T(t)]^T$ and $e(t) = z(t) - z_f(t)$, we can obtain the H_∞ filtering system as follows:

$$\dot{\zeta}(t) = \bar{A}(t)\zeta(t) + \bar{A}_\tau(t)\zeta(t-\tau(t)) + \bar{B}(t)w(t),$$
$$e(t) = \bar{E}(t)\zeta(t) + \bar{E}_\tau(t)\zeta(t-\tau(t)), \qquad (4)$$

where $\zeta(0) = [\chi(t), x_{f0}]$ for $\forall t \in [-\tau_0, 0]$, and

$$\bar{A}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} \phi_i(\psi(t))n_j(g(t)) \begin{bmatrix} A_i & 0 \\ B_{fj}C_i & A_{fj} \end{bmatrix},$$
$$\bar{A}_\tau(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} \phi_i(\psi(t))n_j(g(t)) \begin{bmatrix} A_{\tau i} & 0 \\ B_{fj}C_{\tau i} & 0 \end{bmatrix},$$
$$\bar{B}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} \phi_i(\psi(t))n_j(g(t)) \begin{bmatrix} B_i \\ B_{fj}D_i \end{bmatrix}, \qquad (5)$$
$$\bar{E}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} \phi_i(\psi(t))n_j(g(t))[E_i \ -C_{fj}],$$
$$\bar{E}_\tau(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} \phi_i(\psi(t))n_j(g(t))[E_{\tau i} \ 0].$$

Ergo, the fuzzy H_∞ filter design issue that will be investigated in this paper can be presented as follows:

Fuzzy H_∞ filter issue Design a fuzzy filter in the form of (3) satisfying the following two conditions:

(1) If $w(t) = 0$, the filtering system (4) is asymptotically stable;

(2) For a given scalar $\gamma > 0$, if $\zeta(t) \equiv 0$ for $t \in [-\tau_0, 0]$, the following H_∞ performance can be satisfied for all the $T > 0$ and $w(t) \in L_2[0,\infty)$.

$$\int_0^T \|e(t)\|^2 dt \le \gamma^2 \int_0^T \|w(t)\|^2 dt. \qquad (6)$$

What's more, the following lemma is useful for the later deduction of the main results.

Lemma 1 (Hong et al. [2015]) It is assumed that x is a differentiable function: $[\alpha, \beta] \to \mathbb{R}^n$. For $N_1, N_2, N_3 \in \mathbb{R}^{4n\times n}$, and $R \in \mathbb{R}^{n\times n} > 0$, the following inequality holds:

$$-\int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s)ds \le \xi^T\Omega\xi, \tag{7}$$

where

$$\Omega = \tau(N_1R^{-1}N_1^T + \frac{1}{3}N_2R^{-1}N_2^T + \frac{1}{5}N_3R^{-1}N_3^T) + Sym\{N_1\Delta_1 + N_2\Delta_2 + N_3\Delta_3\},$$
$$d = \beta - \alpha,$$
$$e_i = \begin{bmatrix} 0_{n\times(i-1)n} & I_n & 0_{n\times(4-i)n}\end{bmatrix}, \quad i = 1, 2, 3, 4,$$
$$\Delta_1 = e_1 - e_2, \quad \Delta_2 = e_1 + e_2 - 2e_3,$$
$$\Delta_3 = e_1 - e_2 - 6e_3 + 6e_4$$
$$\xi = [x^T(\beta)\ x^T(\alpha)\ \frac{1}{d}\int_\alpha^\beta x^T(s)ds\ \frac{2}{d^2}\int_\alpha^\beta\int_\alpha^s x^T(u)duds]^T.$$

3. MAIN RESULTS

As fuzzy H_∞ filter has to guarantee the asymptotical stability of the filtering system, we will first analyze the stability condition of system (4).

Lemma 2 For system (4), the constants h, ρ and $\gamma > 0$ are prescribed, it will be asymptotically stable with $w(t) \equiv 0$, and satisfy the H_∞ performance condition (6), if there exist matrices $M = M^T \in \mathbb{R}^{2n\times 2n}$, $N = N^T \in \mathbb{R}^{2n\times 2n}$, $O = O^T \in \mathbb{R}^{2n\times 2n}$, such that the the following inequality is feasible.

$$\Omega(t) = \begin{bmatrix} \Xi + \Theta_3(t) & \sqrt{h}\Gamma_1^T(t)M & \Gamma_2^T(t) \\ * & -MO^{-1}M & 0 \\ * & * & -1 \end{bmatrix} < 0, \tag{8}$$

where

$$\Xi = \frac{3}{h}\begin{bmatrix} -3O & O & 12O & -10O & 0 \\ * & -3O & -8O & 10O & 0 \\ * & * & -64O & 60O & 0 \\ * & * & * & -60O & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

$$\Theta_3(t) = \begin{bmatrix} Sym\{M\bar{A}(t)\} + N & M\bar{A}_\tau(t) & 0 & 0 & M\bar{B}(t) \\ * & -N & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & -\gamma^2 \end{bmatrix},$$

$$\Gamma_1(t) = \begin{bmatrix}\bar{A}(t) & \bar{A}_\tau(t) & 0 & 0 & \bar{B}(t)\end{bmatrix}, \Gamma_2(t) = \begin{bmatrix}\bar{E}(t) & \bar{E}_\tau(t) & 0 & 0 & 0\end{bmatrix}.$$

proof Constructing the Lyapunov-Krasovskii function as follows:

$$V(t) = \varepsilon^T(t)M\varepsilon(t) + \int_{t-\tau(t)}^t \varepsilon^T(s)N\varepsilon(s)ds + \int_{-h}^0\int_{t+\theta}^t \dot{\varepsilon}(s)^TO\dot{\varepsilon}(s)dsd\theta, \tag{9}$$

Differentiating (9) along the trajectories of system (4) yields:

$$\dot{V}(t) = 2\varepsilon^T(t)\dot{\varepsilon}(t) - (1-\dot{\tau}(t))\varepsilon^T(t-\tau(t))N\varepsilon(t-\tau(t)) + h\dot{\varepsilon}(t)^TO\dot{\varepsilon}(t) - \int_{t-h}^t \dot{\varepsilon}(s)^TO\dot{\varepsilon}(s)ds. \tag{10}$$

Applying Lemma 1, we can derive

$$\begin{aligned} &-\int_{t-h}^t \dot{\varepsilon}^T(s)O\dot{\varepsilon}(s)ds < -\int_{t-\tau(t)}^t \dot{\varepsilon}^T(s)O\dot{\varepsilon}(s)ds \\ &\le \xi^T(t)\Big[\tau(t)F_1O^{-1}F_1^T + \frac{\tau(t)}{3}F_2O^{-1}F_2^T \\ &+ \frac{\tau(t)}{5}F_3O^{-1}F_3^T + Sym\{F_1\Pi_1 + F_2\Pi_2 + F_3\Pi_3\}\Big]\xi(t) \\ &< \xi^T(t)\Big[hF_1O^{-1}F_1^T + \frac{h}{3}F_2O^{-1}F_2^T + \frac{h}{5}F_3O^{-1}F_3^T \\ &+ Sym\{F_1\Pi_1 + F_2\Pi_2 + F_3\Pi_3\}\Big]\xi(t) \\ &= \xi^T(t)\left(\Theta_1 + \Theta_2\right)\xi(t), \end{aligned} \tag{11}$$

where

$$\xi(t) = [\varepsilon^T(t)\quad \varepsilon^T(t-\tau(t))\quad \frac{1}{\tau(t)}\int_{t-\tau(t)}^t \varepsilon^T(s)ds\quad \theta_2\quad w(t)]^T,$$
$$\theta_2 = \frac{2}{\tau^2(t)}\int_{t-\tau(t)}^t\int_{t-\tau(t)}^s \varepsilon^T(u)duds,$$
$$\Theta_1 = hF_1O^{-1}F_1^T + \frac{h}{3}F_2O^{-1}F_2^T + \frac{h}{5}F_3O^{-1}F_3^T$$
$$\Theta_2 = Sym\{F_1\Pi_1 + F_2\Pi_2 + F_3\Pi_3\}$$
$$e_i = \begin{bmatrix} 0_{2n\times(i-1)2n} & I_{2n} & 0_{2n\times(5-i)2n}\end{bmatrix}, \quad i = 1, 2, 3, 4, 5,$$
$$\Pi_1 = e_1 - e_2, \quad \Pi_2 = e_1 + e_2 - 2e_3,$$
$$\Pi_3 = e_1 - e_2 - 6e_3 + 6e_4.$$

Based on inequality (2) and equation (4), we can derive

$$\begin{aligned} \dot{V}(t) &< 2\varepsilon^T(t)M[\bar{A}(t)\varepsilon(t) + \bar{A}_\tau(t)\varepsilon(t-\tau(t)) + \bar{B}(t)w(t)] \\ &- (1-\rho)\varepsilon^T(t-\tau(t))N\varepsilon(t-\tau(t)) \\ &+ h\dot{\varepsilon}(t)^TO\dot{\varepsilon}(t) + \xi^T(t)(\Theta_1 + \Theta_2)\xi(t), \end{aligned} \tag{12}$$

and through a straightforward computation we can obtain:

$$\dot{V}(t) + e^T(t)e(t) - \gamma^2w^T(t)w(t) < \xi^T(t)(\Theta_1 + \Theta_2 + \Theta_3(t) + h\Gamma_1^TO\Gamma_1(t) + \Gamma_2^T\Gamma_2(t))\xi(t), \tag{13}$$

where $\Theta_3(t)$, $\Gamma_1(t)$, $\Gamma_2(t)$ are defined in (8).

From the inequality (13), it can be inferred that if

$$\Theta_1 + \Theta_2 + \Theta_3(t) + h\Gamma_1^TO\Gamma_1(t) + \Gamma_2^T\Gamma_2(t) < 0, \tag{14}$$

the following inequality holds

$$\dot{V}(t) + e^T(t)e(t) - \gamma^2w^T(t)w(t) < 0. \tag{15}$$

Further, we can derive

$$\int_0^L (\|e(t)\|^2 - \gamma^2\|w(t)\|^2)dt + V(t)|_{t=L} - V(t)|_{t=0} \le 0, \tag{16}$$

for $V(t)|_{t=0} = 0$, and $V(t)|_{t=L} \ge 0$. Inequality (16) can be easily converted to inequality (6), which implies the H_∞ performance requirement is satisfied.

Moreover, to reduce computational complexity, we assume

$$\begin{aligned} F_1 &= \frac{1}{h}\left[-O\ O\ 0\ 0\ 0\right]^T, \\ F_2 &= \frac{3}{h}\left[-O\ -O\ 2O\ 0\ 0\right]^T, \\ F_3 &= \frac{5}{h}\left[-O\ O\ 6O\ -6O\ 0\right]^T. \end{aligned} \tag{17}$$

Thus, $\Theta_1 + \Theta_2$ can be denoted as Ξ, where Ξ is defined in (8).

Using the Schur Complement criterion, the inequality (14) is equivalent to (8). Then following similar deduction process, in the light of the inequalities (14), (13), we can derive $\dot{V}(t) < 0$ when $w(t) \equiv 0$, which means the filtering system (4) is asymptotically stable. Hence the proof of Lemma 2 is finished.

Remark 1 From proof of Lemma 2, we can see that a new integral inequality (7) is used to estimate the upper bound of integral term $-\int_{t-h}^{t} \dot{\varepsilon}^T(s) O \dot{\varepsilon}(s) ds$, which is normally dealt with by the inequalities derived from the Leibniz-Newton formula. Since the novel integral inequality is tighter than those derived from the Leibniz-Newton formula, less conservative results can be achieved. Besides, it can be seen that the LMIs (8) derived from (7) are relatively simple, which means the implementation cost could be lowered.

Theorem 1 Given constants h, ρ, υ and $\gamma > 0$, the system (4) is asymptotically stable with $w(t) \equiv 0$, and satisfies the H_∞ performance condition (6), if there exist matrices

$$\tilde{M} = \begin{bmatrix} M_{11} & \tilde{M}_{22} \\ * & \tilde{M}_{22} \end{bmatrix}, \quad (18)$$

$\tilde{N} = \tilde{N}^T \in \mathbb{R}^{2n\times 2n}$, $\tilde{O} = \tilde{O}^T \in \mathbb{R}^{2n\times 2n}$, such that the following LMIs are feasible.

$$\tilde{\Omega}_{ij} = \begin{bmatrix} \tilde{\Theta}_{3ij} + \tilde{\Xi} & \sqrt{h}\tilde{\Gamma}_{1ij}^T & \tilde{\Gamma}_{2ij}^T \\ * & -2\upsilon\tilde{M} + \upsilon^2\tilde{O}_i & 0 \\ * & * & -1 \end{bmatrix} < 0 \quad (19)$$

where

$$\tilde{\Theta}_{3ij} = \begin{bmatrix} Sym\{\lambda_{1ij}\} + \tilde{N} & \lambda_{2ij} & 0 & 0 & \lambda_{3ij} \\ * & -\tilde{N} & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & -\gamma^2 \end{bmatrix},$$

$$\tilde{\Xi} = \frac{3}{h}\begin{bmatrix} -3\tilde{O} & \tilde{O} & 12\tilde{O} & -10\tilde{O} & 0 \\ * & -3\tilde{O} & -8\tilde{O} & 10\tilde{O} & 0 \\ * & * & -64\tilde{O} & 60\tilde{O} & 0 \\ * & * & * & -60\tilde{O} & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_{1ij} = [\lambda_{1ij} \ \ \lambda_{2ij} \ \ 0 \ \ 0 \ \ \lambda_{3ij}], \tilde{\Gamma}_{2ij} = \left[[E_i \ \ -\mathscr{C}_j] \ \ [E_{\tau i} \ \ 0] \ \ 0 \ \ 0 \ \ 0 \right],$$

$$\lambda_{1ij} = \begin{bmatrix} M_{11}A_i + \mathscr{B}_j C_i & \mathscr{A}_j \\ \tilde{M}_{22}A_i + \mathscr{B}_j C_i & \mathscr{A}_j \end{bmatrix}, \lambda_{2ij} = \begin{bmatrix} M_{11}A_{\tau i} + \mathscr{B}_j C_{\tau i} & 0 \\ \tilde{M}_{22}A_{\tau i} + \mathscr{B}_j C_{\tau i} & 0 \end{bmatrix},$$

$$\lambda_{3ij} = \begin{bmatrix} M_{11}B_i + \mathscr{B}_j D_i \\ \tilde{M}_{22}B_i + \mathscr{B}_i D_i \end{bmatrix}, i = 1, 2, ..., p, j = 1, 2, ..., c,$$

and in this case, the parameters of the fuzzy filter can be presented as

$$A'_{fj} = \tilde{M}_{22}^{-1}\mathscr{A}_j, \quad B'_{fj} = \tilde{M}_{22}^{-1}\mathscr{B}_j, \quad C'_{fj} = \mathscr{C}_j. \quad (20)$$

proof For any scalar υ, the following inequality is true.

$$(\upsilon O - M)O^{-1}(\upsilon O - M) \geq 0, \quad (21)$$

which can also be denoted as

$$-MO^{-1}M \leq -2\upsilon M + \upsilon^2 O. \quad (22)$$

Consequently, if the inequality (23) holds, the inequality (8) will be true.

$$\Omega_1(t) = \begin{bmatrix} \Xi + \Theta_3(t) & \sqrt{h}\Gamma_1^T M & \Gamma_2^T(t) \\ * & -2\upsilon M + \upsilon^2 O & 0 \\ * & * & -1 \end{bmatrix} < 0. \quad (23)$$

Then we'll introduce a partition as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix}, \quad (24)$$

where $M_{11} > 0$, $M_{22} > 0$, and M_{12} is assumed to be invertible via invoking small perturbation if it is necessary.

Assume

$$\mathscr{S} = \begin{bmatrix} I & 0 \\ * & M_{22}^{-T}M_{12}^T \end{bmatrix}, \quad (25)$$

and $\mathscr{R} = diag\{\mathscr{S}, \mathscr{S}, \mathscr{S}, \mathscr{S}, 1\}$. Pre and post multiplying (23) with $diag\{\mathscr{R}, \mathscr{S}, 1\}$, then we can get

$$\tilde{\Omega}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} \phi_i(\psi(t)) n_j(g(t)) \tilde{\Omega}_{ij}, \quad (26)$$

with the changes of variables as

$$\begin{aligned} &\tilde{M}_{22} = M_{12}M_{22}^{-1}M_{12}^T, \tilde{M} = \mathscr{S}^T M \mathscr{S} = \begin{bmatrix} M_{11} & \tilde{M}_{22} \\ * & \tilde{M}_{22} \end{bmatrix} \\ &\tilde{Y} = \mathscr{S}^T Y \mathscr{S}, \tilde{O}(t) = \mathscr{S}^T O \mathscr{S}, \mathscr{A}_j = M_{12}A_{fj}M_{22}^{-T}M_{12}^T, \\ &\mathscr{B}_j = M_{12}B_{fj}, \mathscr{C}_j = C_{fj}M_{22}^{-T}M_{12}^T, \end{aligned} \quad (27)$$

where $\tilde{\Omega}_{ij}$ is defined in (19).

Therefore, we can derive that (8) holds, if $\tilde{\Omega}_{ij} < 0$ is true. And according to Lemma 2, we know that $\Omega(t) < 0$ means the filtering system (4) is asymptotically stable and satisfy the H_∞ performance condition (6).

Besides, from (27), we can obtain:

$$\begin{aligned} &A_{fj} = M_{12}^{-1}\mathscr{A}_j M_{12}^{-T}M_{22}^T, \quad B_{fj} = M_{12}^{-1}\mathscr{B}_j, \\ &C_{fj} = \mathscr{C}_j M_{12}^{-T}M_{22}^T. \end{aligned} \quad (28)$$

As $\tilde{M}_{22} = M_{12}M_{22}^{-1}M_{12}^T$, through an equivalent transformation $M_{12}^{-T}M_{22}x_f(t)$, we can obtain an admissible fuzzy H_∞ realization as:

$$\begin{aligned} &A'_{fj} = M_{12}^{-T}M_{22}(M_{12}^{-1}\mathscr{A}_j M_{12}^{-T}M_{22}^T)M_{22}^{-T}M_{12}^T = \tilde{M}_{22}^{-1}\mathscr{A}_j, \\ &B'_{fj} = M_{12}^{-T}M_{22}(M_{12}^{-1}\mathscr{B}_j) = \tilde{M}_{22}^{-1}\mathscr{B}_j, \\ &C'_{fj} = (\mathscr{C}_j M_{12}^{-T}M_{22}^T)M_{22}^{-T}M_{12}^T = \mathscr{C}_j. \end{aligned} \quad (29)$$

Hence, the proof of Theorem 1 is completed.

Theorem 2 Given constants h, ρ, υ and $\gamma > 0$, the system (4) is asymptotically stable with $w(t) \equiv 0$, and satisfies the H_∞ performance condition (6), if there exist matrices

$$\tilde{M} = \begin{bmatrix} M_{11} & \tilde{M}_{22} \\ * & \tilde{M}_{22} \end{bmatrix} > 0, \quad (30)$$

$\tilde{N} = \tilde{N}^T \in \mathbb{R}^{2n\times 2n} > 0$, $\tilde{O} = \tilde{O}^T \in \mathbb{R}^{2n\times 2n} > 0$, $M_{ij} = M_{ij}^T \in \mathbb{R}^{(10n+2)\times(10n+2)}$, such that the following LMIs are feasible.

$$\begin{aligned} &\tilde{\Omega}_{ij} - M_{ij} + Q_{ij} + \sum_{r=1}^{p}\sum_{s=1}^{c} \bar{d}_{ij}M_{rs} - \sum_{a=1}^{p}\sum_{b=1}^{c} \underline{d}_{ij}M_{ab} < 0, \\ &i = 1, 2, ..., p, j = 1, 2, ..., c, \end{aligned} \quad (31)$$

where $\tilde{\Omega}_{ij}$ is defined in (19). And in this case, the parameters of the fuzzy filter can also be presented as (20).

proof In this part, for the convenience of notations, we denote

$$\phi_i(\psi(t)) = \phi_i, \quad n_j(g(t)) = n_j, \quad \phi_i(\psi(t))n_j(g(t)) = d_{ij}, \tag{32}$$

and assume $\underline{d}_{ij}$ and $\bar{d}_{ij}$ are the lower bound and upper bound of d_{ij}, respectively.

From Lemma 2 and Theorem 1, we have

$$\dot{V}(t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) < \xi^T(t)\tilde{\Omega}(t)\xi(t), \tag{33}$$

so we can derive

$$\begin{aligned}
\xi^T(t)\tilde{\Omega}(t)\xi(t) &= \sum_{i=1}^{p}\sum_{j=1}^{c} d_{ij}\xi^T(t)\Omega_{ij}\xi(t) \\
&\le \sum_{i=1}^{p}\sum_{j=1}^{c} d_{ij}\xi^T(t)\Omega_{ij}\xi(t) + \sum_{i=1}^{p}\sum_{j=1}^{c}(\bar{d}_{ij} - d_{ij})\xi^T(t)M_{ij}\xi(t) \\
&\quad + \sum_{i=1}^{p}\sum_{j=1}^{c}(d_{ij} - \underline{d}_{ij})\xi^T(t)Q_{ij}\xi(t) \\
&= \sum_{i=1}^{p}\sum_{j=1}^{c} d_{ij}\xi^T(t)(\Omega_{ij} - M_{ij} + Q_{ij} + \sum_{r=1}^{p}\sum_{s=1}^{c}\bar{d}_{rs}M_{rs} \\
&\quad - \sum_{a=1}^{p}\sum_{b=1}^{c}\underline{d}_{ab}Q_{ab})\xi(t).
\end{aligned} \tag{34}$$

Consequently, if LMIs (31) holds, we can get $\tilde{\Omega}(t) < 0$, which means both the H_∞ performance condition (6) and the asymptotically stable requirement can be satisfied. Thus, the proof of Theorem 2 is finished.

Remark 2

It can be seen that the information of the membership functions is considered in Theorem 2 while Theorem 1 is membership functions independent. As a result, Theorem 2 is less conservative than Theorem 1. However, Theorem 2 also includes complex matrices M_{ij}, Q_{ij}, $(i = 1, ...p, j = 1, ..., c)$, which means it will be more difficult to be realized in engineering applications. Therefore, both Theorem 1 and Theorem 2 are meaningful.

4. SIMULATION

In this section, two simulation examples will be provided to demonstrate the effectiveness and superiority of the designed criteria.

4.1 Example 1

Consider the example given in Lin et al. [2008], which can be presented as (1) with

$$A_1 = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, A_{\tau 1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix},$$

$$A_{\tau 2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix},$$

$$C_1 = [\ 1\ 0], C_2 = [\ 0.5\ -0.6], C_{\tau 1} = [\ -0.8\ 0.6],$$

$$C_{\tau 2} = [\ -0.2\ 1], D_1 = 0.3, D_2 = -0.6, E_1 = [\ 1\ -0.5],$$

$$E_2 = [\ -0.2\ 0.3], E_{\tau 1} = [\ 0.1\ 0], E_{\tau 2} = [\ 0\ 0.2].$$

and the membership functions of the fuzzy model and the fuzzy filter are chosen as

$$\phi_1(\psi(t)) = 1 - \frac{0.5}{1 + e^{-3-t}}, \quad \phi_2(\psi(t)) = 1 - \phi_1(\psi(t)),$$

$$n_1(g(t)) = 0.7 - \frac{0.5}{1 + e^{4-t}}, \quad n_2(g(t)) = 1 - n_1(g(t)).$$

Let $(\rho, \upsilon, h) = (0.2, 1, 0.5)$, using the LMIs (31) presented in Theorem 2, the minimum attenuation level $\gamma = 0.18$ can be obtained. Moreover, according to the feasible solutions to LMIs (31), and using (20), we can get the parameters of the fuzzy H_∞ filter as:

$$A_{f1} = \begin{bmatrix} -2.7979 & 0.1951 \\ 0.6809 & -2.2651 \end{bmatrix}, A_{f2} = \begin{bmatrix} -2.3839 & 0.057 \\ -0.2657 & -1.7317 \end{bmatrix},$$

$$B_{f1} = \begin{bmatrix} -0.9518 \\ 0.4154 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.9386 \\ 0.2307 \end{bmatrix},$$

$$C_{f1} = [-0.6254\ 0.3172], C_{f2} = [-0.2457\ 0.0704].$$

To fully demonstrate the validity of the proposed method, we will use Theorem 2 of this paper and some other recently developed fuzzy H_∞ filter design methods, respectively to find the minimum attenuation level γ. The corresponding results are listed in Table 1-3.

Table 1. The minimum attenuation level γ for $\upsilon = 2$

method	$h = 0.5$	$h = 0.6$	$h = 0.8$	$h = 1$
Lin et al. [2008]	0.35	0.36	0.38	0.41
Su et al. [2009]	0.25	0.25	0.27	0.29
Zhang et al. [2009]	0.25	0.25	0.27	0.29
Huang et al. [2011]	0.24	0.25	0.25	0.26
Zhou and He [2015]	0.23	0.24	0.25	0.25
Th. 2	0.18	0.18	0.18	0.18

Table 2. The minimum attenuation level γ for $\upsilon = 5$

method	$h = 0.5$	$h = 0.6$	$h = 0.8$	$h = 1$
Lin et al. [2008]	0.34	0.34	0.35	0.37
Su et al. [2009]	0.24	0.24	0.25	0.26
Zhang et al. [2009]	0.24	0.24	0.25	0.26
Huang et al. [2011]	0.24	0.24	0.25	0.26
Zhou and He [2015]	0.23	0.24	0.24	0.25
Th. 2	0.17	0.17	0.18	0.18

Table 3. The minimum attenuation level γ for $\upsilon = 20$

method	$h = 0.5$	$h = 0.6$	$h = 0.8$	$h = 1$
Lin et al. [2008]	0.37	0.45	1.01	$--$
Su et al. [2009]	0.26	0.32	0.70	$--$
Zhang et al. [2009]	0.26	0.28	0.44	$--$
Huang et al. [2011]	0.25	0.26	0.35	0.45
Zhou and He [2015]	0.23	0.24	0.25	0.25
Th. 2	0.17	0.17	0.17	0.17

where $--$ denotes that the minimum attenuation level γ does not exist.

From Table 1-3, we can clearly see that smaller minimum attenuation level γ can be obtained with Theorem 2 than the ones obtained with other methods, which implies that the method proposed in this paper is less conservative than those in (Huang et al. [2011], Lin et al. [2008], Su et al. [2009], Zhang et al. [2009], Zhou and He [2015]).

Remark 4 The less conservative results can be obtained with the approach proposed in this paper mainly because of two reasons. First, the new integral inequality (7) is employed to derive stability condition, which is tighter than those derived from the Leibniz-Newton formula. Besides, theorem 2 in this paper has fully considered the information about the membership functions while the methods in other literature are membership functions independent.

4.2 Example 2

Consider the example given in Su et al. [2009], which can be presented as (1) with

$$A_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -0.9 & 0 \\ 0 & -0.5 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} -0.9 & 0.2 & 0 \\ -0.2 & -0.5 & 0 \\ 0 & -0.1 & -0.8 \end{bmatrix},$$

$$A_{\tau 1} = \begin{bmatrix} -0.8 & 0.2 & -0.1 \\ 0.1 & -0.8 & 0 \\ -0.4 & 0.25 & -1 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} -1 & 0.5 & 0.1 \\ 0.5 & -1 & 0 \\ -0.8 & 0.9 & -0.25 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix}, C_1 = [0.5\ 0.4\ 0], C_2 = [0.5\ -1\ 0],$$

$$C_{\tau 1} = [1\ -0.5\ 0.5], C_{\tau 2} = [0.1\ 0.5\ 0.5],$$

$$D = 0.25, \quad E_1 = [0.5\ 0\ 0], E_2 = [1\ -0.5\ 0],$$

$$E_{\tau 1} = [0.1\ 0.5\ 0.5], E_{\tau 2} = [0.1\ 0\ 0.5].$$

In this example, we choose the same membership functions as in Example 1. Similarly, we use Theorem 2 and other filter design methods, respectively to find the minimum attenuation level γ, the results are listed in Table III-IV.

Table 4. The minimum attenuation level γ for $\rho = 0.2$, $h = 0.6$

method	$\omega = 0.7$	$\omega = 1$	$\omega = 2$	$\omega = 4$
Su et al. [2009]	0.31	0.27	0.27	0.29
Huang et al. [2011]	0.28	0.27	0.27	0.28
Th. 3	0.17	0.15	0.16	0.20

Table 5. The minimum attenuation level γ for $\rho = 0.2$, $h = 0.8$

method	$\omega = 0.7$	$\omega = 1$	$\omega = 2$	$\omega = 4$
Su et al. [2009]	0.45	0.34	0.31	0.39
Huang et al. [2011]	0.33	0.31	0.31	0.31
Th. 3	0.26	0.20	0.20	0.25

It can be seen that Theorem 2 can yield smaller minimum attenuation level γ than Huang et al. [2011], Su et al. [2009], which implies that the proposed method in this paper is more relaxed than those in Huang et al. [2011], Su et al. [2009].

5. CONCLUSIONS

In this paper, the fuzzy H_∞ filter design problem has been investigated for nonlinear time-delay systems. The T-S fuzzy model has been used to describe the dynamics of the system, and two LMI-based criteria have been derived. Unlike conventional fuzzy H_∞ filter, The designed fuzzy filter has been allowed to freely choose the premise membership functions and the number of rules, ergo, robustness to uncertainty and lower implementation cost can be realized. Besides, to reduce the conservatism of the derived results, a novel integral inequality which is tighter than other existing ones has been introduced, and the information of the membership functions have been taken into account. Finally, two examples have demonstrated the validity of the designed fuzzy H_∞ filter.

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